RANGES OF ALLOWABLE COMPONENT VALUES FOR THE SYNTHESIS OF SPECIFIED RC TRANSFER FUNCTIONS

JACK WARNER LILLIS

/0

RANGES OF ALLOWABLE COMPONENT VALUES FOR THE SYNTHESIS OF SPECIFIED RC TRANSFER FUNCTIONS

by

Jack Warner Lillis
Lieutenant, United States Navy
B.S.E.E., Purdue University, 1961

[redacted]

Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN ENGINEERING ELECTRONICS

from the

NAVAL POSTGRADUATE SCHOOL
June 1967

ABSTRACT

The theory of continuants is applied to the analysis of general ladder networks of the first Cauer form to provide a concise, compact, and readily calculable form for the driving point and transfer functions used to describe the network. A new procedure is established for the synthesis of RC ladder networks up to nth order from a given voltage-ratio transfer function. Ranges of values for the network components are shown to exist. The procedure is readily adaptable for use with a digital computer. Two programs written in FORTRAN 63 language are provided for a third order system to illustrate how the procedure can be used with a digital computer for computer-aided design of the networks. Output of the programs is in the form of graphs and tables of component values.

TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF ILLUSTRATIONS

ACKNOWLEDGEMENT

I wish to express my appreciation for the assistance and encouragement given me by Dr. S. R. Parker in the writing of this thesis.

1. Introduction.

Driving point functions of ladder networks are frequently used in the analysis and synthesis of these networks. The input impedance and/ or input admittance can be readily expressed in the form of a continued fraction. Many good texts have been written which thoroughly explore the many facets of the problem. Recently the application of the theory of continuants to ladder networks has been receiving renewed interest, as evidenced by a number of papers in the current literature. Continuants provide a convenient, concise, and readily calculable form for deriving and expressing results of network analysis.

It is often of interest to view the ladder network as a two-port system. If we impress a driving function at the input terminals, we generally want to know what we will get at the output terminals. Conversely, we might wish a particular response from a given driving function. The task is then to find a network which will give that response. The problem of synthesis is generally more difficult than that of analysis.

This paper shall be concerned with the synthesis of RC ladder networks of the first Cauer form from a given voltage-ratio transfer function. Continuants are used in the analysis of general ladder networks to illustrate their usefulness in expressing transfer and driving point functions in concise and compact forms. A procedure is developed for synthesizing networks up to nth order. It is shown that there are many sets of component values which can be used to build a suitable network. Two programs have been written and included to illustrate how one can use a digital computer to great advantage in the synthesis problem.

2. Continued Fractions.

A continued fraction is an expression of the form

$$a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{a_3 + \cdots}} \tag{2.1}$$

which may be written in a more compact form with dropped +'s as

$$a_1 + \frac{b_1}{a_2 +}\ \frac{b_2}{a_3 +}\ \frac{b_3}{a_4 +} \cdots\cdots \tag{2.2}$$

In this paper only continued fractions whose b terms are unity will be considered.

A terminating continued fraction is one which has a finite number of denominators as opposed to an infinite continued fraction which has an infinite number of denominators. A continued fraction terminating in a finite number of denominators defines a convergent of the continued fraction. This convergent is usually expressed as a ratio p/q. In the continued fraction

$$a_1 + \frac{1}{a_2 +}\ \frac{1}{a_3 +}\ \frac{1}{a_4 + \cdots\cdot} \tag{2.3}$$

$\frac{p_1}{q_1} = a_1$ is called the first convergent;

$\frac{p_2}{q_2} = a_1 + \frac{1}{a_2} = \frac{a_1 a_2 + 1}{a_2}$ is called the second convergent;

$\frac{p_3}{q_3} = a_1 + \frac{1}{a_2 +}\ \frac{1}{a_3} = \frac{a_1 a_2 a_3 + a_1 + a_3}{a_2 a_3 + 1}$ is called the third convergent; and so on. The nth convergent is given by

$$\frac{p_n}{q_n} = a_1 + \frac{1}{a_2 +}\,\frac{1}{a_3 +}\,\cdots\,\frac{1}{a_n} .$$

The following properties of continued fractions and convergents, taken from the text by Wells,[1] are submitted without proof.

1) Any ordinary fraction in its lowest terms may be converted into a terminating continued fraction.

2) The difference between two consecutive convergents $\frac{p_n}{q_n}$ and $\frac{p_{n+1}}{q_{n+1}}$ is $\frac{1}{q_n\, q_{n+1}}$

3) The even convergents are greater, and the odd convergents less than the fraction itself.

4) Any convergent is nearer than the preceding convergent to the value of the fraction itself.

3. <u>Continuants.</u>

The theory of continued fractions is greatly simplified by means of a class of rational functions defined by a sequence of linear equations. These functions called <u>Continuants</u> were originally due to Euler. A good discussion of continuants appears in the text by Bartlett. [1]

The rational integral function, p_n, of the n quantities a_1, a_2, a_3, ----a_n is defined by the set of equations

$$p_0 = 1$$

$$p_1 = a_1$$

$$p_2 = a_2\, p_1 + p_0$$

[1]W. Wells, Advanced Course in Algebra (New York: D. C. Heath and Company, 1904), pp. 449-456.

$$p_3 = a_3\, p_2 + p_1 \tag{3.1}$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$p_n = a_n\, p_{n-1} + p_{n-2}$$

where the coefficients a_1, a_2, a_3, --a_n and the first two functions, p_0 and p_1, are given. The function p_n thus defined is termed a "Simple Continuant of the nth Order", and is denoted by $k(a_1, a_2, a_3, -- a_n)$ which may be further shortened to $k(a_1, a_n)$ for the sake of brevity by using only the first and last arguments of the function in the notation.

Simple continuants are then defined by the set of equations

$$p_0 = k(0) = 1$$

$$p_1 = k(a_1) = a_1 \tag{3.2}$$

$$p_2 = k(a_1, a_2) = a_2\, p_1 + p_0 = a_2\, k(a_1) + k(0)$$

$$p_3 = k(a_1, a_3) = a_3\, p_2 + p_1 = a_3\, k(a_1, a_2) + k(a_1)$$

$$\cdot$$
$$\cdot$$
$$\cdot$$
$$\cdot$$

$$p_n = k(a_1, a_n) = a_n\, p_{n-1} + p_{n-2} = a_n\, k(a_1, a_{n-1}) + k(a_1, a_{n-2})$$

Each succeding function, therefore, has been defined in terms of a linear combination of the two preceding functions with the exception of p_0 and p_1. It is important to note that the value of the zero order continuant, $k(0)$, is unity. If the coefficient a_1 has the value of zero, then the first order continuant $k(a_1)$ also has the value of zero and should be written as $k(a_1 = 0) = 0$ in order to avoid confusion.

In general, if the first argument of the function is a_r, the continuant for $n > r$ shall be written as

$$p_n = k(a_r, a_n) = a_n k(a_r, a_{n-1}) + k(a_r, a_{n-2})$$

where p_n is defined by the equations

$$\begin{aligned}
p_{r-1} &= 1 = k(a_r, a_{r-1}) \\
p_r &= a_r = k(a_r, a_r) \\
p_{r+1} &= a_{r+1} p_r + p_{r-1} = k(a_r, a_{r+1}) = a_{r+1} k(a_r, a_r) \\
&\quad + k(a_r, a_{r-1}) \\
&\vdots \\
p_n &= a_n p_{n-1} + p_{n-2} = k(a_r, a_n) = a_n k(a_r, a_{n-1}) \\
&\quad + k(a_r, a_{n-2})
\end{aligned} \tag{3.3}$$

A second rational integral function, q_n, may be defined in a similar fashion such that q_2 is the same function of a_2 as p_1 is of a_1; q_3 is the same function of a_2, a_3 as p_2 is of a_1, a_2; q_n is the same function of a_2, a_3---a_n as p_{n-1} is of a_1, a_2---a_{n-1}; etc.. These q's are also continuants and may be written as

$$\begin{aligned}
q_1 &= k(0) = 1 \\
q_2 &= k(a_2) = a_2 \\
q_3 &= k(a_2, a_3) = a_3 q_2 + q_1 = a_3 k(a_2) + k(0) \\
&\vdots \\
q_n &= k(a_2, a_n) = a_n q_{n-1} + q_{n-2} = a_n k(a_2, a_{n-1}) \\
&\quad + k(a_2, a_{n-2})
\end{aligned} \tag{3.4}$$

In relating the continuants which have just been defined to the successive convergents of a continued fraction, the following expressions can be written:

$$a_1 = \frac{p_1}{q_1} = \frac{k(a_1)}{k(0)}$$

$$a_1 + \frac{1}{a_2} = \frac{p_2}{q_2} = \frac{k(a_1, a_2)}{k(a_2)} \tag{3.5}$$

$$a_1 + \frac{1}{a_2 +} \; \frac{1}{a_3 +} \; \ldots \frac{1}{a_n} = \frac{p_n}{q_n} = \frac{k(a_1, a_n)}{k(a_2, a_n)}$$

Expansion of a continuant of fairly high order by means of the fundamental definitions may be a long process. Several different techniques for expanding continuants have been devised to decrease the amount of work required. Bartlett presents a systematic technique based on a schematic arrangement due to Hindenburg (1741-1808). [1] Parker, Peskin, and Chirlian[2] have shown a means of representing a continuant as a signal-flow graph. Expansion of the continuant is accomplished by evaluating the signal-flow graph.

An alternate expression for the simple continuant $k(a_1, a_n)$ is given the Sylvester-Muir determinant:

[1]A. C. Bartlett, <u>The Theory of Electrical Artificial Lines and Filters</u> (New York: J. Wiley and Sons, 1931), pp. 45-46.

[2]S. R. Parker, E. Peskin, and P. M. Chirlian, "Continuants, Signal Flow Graphs, and Ladder Networks", <u>Proceedings of the IEEE</u>, Vol 54, March 1966, pp. 422-423.

$$k(a_1, a_n) = \begin{vmatrix} a_1 & +1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -1 & a_2 & +1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & a_3 & +1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -1 & a_4 & +1 & 0 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -1 & a_{n-1} & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 & a_n \end{vmatrix}$$

Expansion of the determinant yields the expansion of the continuant.

The total number of terms in the expansion of an nth order continuant is given by the expression

$$1 + (n-1) + \frac{(n-2)\ (n-3)}{2!} + \frac{(n-3)\ (n-4)\ (n-5)}{3!} + \ldots.. \tag{3.6}$$

which can be proved equal to

$$\frac{(1 + \sqrt{5})^{n+1} - (1 - \sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}} \tag{3.7}$$

Results of these expressions can be used as checks on the expansions.

The continuant expansions of

$$Q_m(s) = k(R_1, sC_n)$$

for values of m up to m=8 are given in Appendix IV. These will be used in subsequent sections of this paper.

4. <u>Continuants Applied to A General Ladder Network.</u>

Let Fig. 1 represent a general ladder network constructed of admittances Y_2, Y_4, Y_6,...Y_n and impedances Z_1, Z_3, Z_5, ... Z_{n-1}.

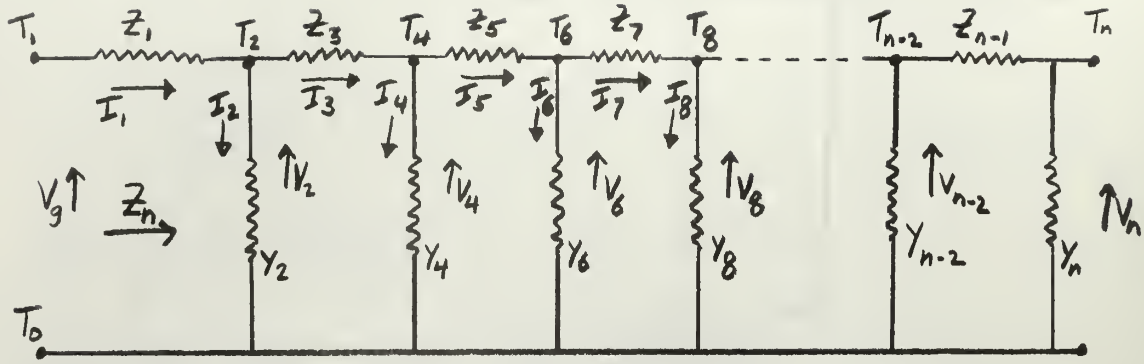


Fig. 1. General Ladder Network

The impedance, Z_n, of the network looking into terminals T_0T_1 can be written as a continued fraction by use of the rules for combining series impedances and shunt admittances. This impedance is given by

$$Z_n = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cdots + \cfrac{1}{Y_{n-2} + \cfrac{1}{Z_{n-1} + \cfrac{1}{Y_n}}}}}} \tag{4.1}$$

which can be expressed as a ratio of continuants as

$$Z_n = \frac{k(Z_1,Y_n)}{k(Y_2,Y_n)} \tag{4.2}$$

Input current, I_1, to the ladder network is given by

$$I_1 = \frac{V_g}{Z_n} = V_g \left[\frac{k(Y_2,\ Y_n)}{k(Z_1,\ Y_n)}\right] \tag{4.3}$$

Current I_3 is that fraction of I_1 given by

$$I_3 = I_1 \left[\frac{\text{Admittance of network } (Z_3, Y_4, Z_5, Y_6 \ldots Z_{n-1}, Y_n)}{\text{Admittance of network } (Y_2, Z_3, Y_4, Z_5 \ldots Z_{n-1}, Y_n)}\right] \quad (4.4a)$$

$$= I_1 \left[\frac{\dfrac{k(Y_4, Y_n)}{k(Z_3, Y_n)}}{Y_2 + \dfrac{k(Y_4, Y_n)}{k(Z_3, Y_n)}}\right] = I_1 \left[\frac{k(Y_4, Y_n)}{Y_2\, k(Z_3, Y_n) + k(Y_4, Y_n)}\right] \quad (4.4b)$$

Using theorem 2 of Appendix I, this further reduces to

$$I_3 = I_1 \left[\frac{k(Y_4, Y_n)}{k(Y_2, Y_n)}\right] \quad (4.4c)$$

In a similar fashion

$$I_5 = I_3 \left[\frac{k(Y_6, Y_n)}{k(Y_4, Y_n)}\right] = I_1 \left[\frac{k(Y_4, Y_n)}{k(Y_2, Y_n)} \times \frac{k(Y_6, Y_n)}{k(Y_4, Y_n)}\right] \quad (4.5a)$$

$$= I_1 \left[\frac{k(Y_6, Y_n)}{k(Y_2, Y_n)}\right] \quad (4.5b)$$

In general, the current I_{2r+1} through any series impedance Z_{2r+1} can be related to the input current I_1 as

$$I_{2r+1} = I_1 \left[\frac{k(Y_{2r+2}, Y_n)}{k(Y_2, Y_n)}\right] \quad (4.6)$$

by a continuation of the above process. The ratio of current I_{2r+1} through Z_{2r+1} to current I_{2m+1} through Z_{2m+1} is given by

$$\frac{I_{2r+1}}{I_{2m+1}} = \left[\frac{I_1 \dfrac{k(Y_{2r+2}, Y_n)}{k(Y_2, Y_n)}}{I_1 \dfrac{k(Y_{2m+2}, Y_n)}{k(Y_2, Y_n)}}\right] = \left[\frac{k(Y_{2r+2}, Y_n)}{k(Y_{2m+2}, Y_n)}\right] \tag{4.7}$$

The voltages across the shunt admittances should be considered next. The voltage V_2 is given by

$$V_2 = V_g \left[\frac{\text{Impedance of network } (Y_2, Z_3, Y_4, Z_5, \ldots . Z_{n-1}, Y_n)}{Z_n}\right] \tag{4.8a}$$

$$= V_g \left[\frac{k(Z_3, Y_n)}{k(Y_2, Y_n)} \div \frac{k(Z_1, Y_n)}{k(Y_2, Y_n)}\right] \tag{4.8b}$$

$$= V_g \left[\frac{k(Z_3, Y_n)}{k(Z_1, Y_n)}\right] \tag{4.8c}$$

In similar fashion the voltage V_4 is given by

$$V_4 = V_2 \left[\frac{k(Z_5, Y_n)}{k(Z_3, Y_n)}\right] = V_g \left[\frac{k(Z_3, Y_n)}{k(Z_1, Y_n)} \times \frac{k(Z_5, Y_n)}{k(Z_3, Y_n)}\right] \tag{4.9a}$$

$$= V_g \left[\frac{k(Z_5, Y_n)}{k(Z_1, Y_h)}\right] \tag{4.9b}$$

In general, the voltage V_{2r} across shunt admittance Y_{2r} can be related to the input voltage V_g as

$$V_{2r} = V_g \left[\frac{k(Z_{2r+1}, Y_n)}{k(Z_1, Y_n)}\right] \tag{4.10}$$

The ratio of voltage V_{2r} across shunt admittance Y_{2r} to voltage V_{2m} across shunt admittance Y_{2m} is given by

$$\frac{V_{2r}}{V_{2m}} = \left[\frac{k(Z_{2r+1}, Y_n)}{k(Z_{2m+1}, Y_n)}\right] \tag{4.11}$$

It is often of interest to determine the voltage-ratio transfer function, V_n/V_g, of the ladder network. This can be done in the following manner:

letting

$$2r+1 = n-1$$

then

$$n = 2r+2$$

Using Eq. (4.6), we get

$$I_{n-1} = I_1 \left[\frac{k(Y_n, Y_n)}{k(Y_2, Y_n)}\right] \tag{4.12}$$

Since

$$I_1 = V_g/Z_n \tag{4.13}$$

then

$$V_n = \frac{I_{n-1}}{Y_n} = \left[\frac{k(Y_n, Y_n)}{Y_n k(Y_2, Y_n)} \times \frac{V_g}{Z_n}\right] \tag{4.14}$$

and substituting from Eq. (4.2) for Z_n gives

$$\frac{V_n}{V_g} = \left[\frac{k(Y_n, Y_n)}{Y_n k(Y_2, Y_n)} \times \frac{k(Y_2, Y_n)}{k(Z_1, Y_n)}\right] = \left[\frac{k(Y_n, Y_n)}{Y_n k(Z_1, Y_n)}\right] \tag{4.15}$$

but

$$k(Y_n, Y_n) = Y_n$$

therefore

$$\frac{V_n}{V_g} = \frac{1}{k(Z_1, Y_n)} \tag{4.16}$$

If the ladder network is such that $Z_1 = 0$, then using Theorem 6 of Appendix I gives

$$Z_n = \frac{k(Z_3, Y_n)}{k(Y_2, Y_n)} \tag{4.17}$$

$$\frac{V_n}{V_g} = \frac{1}{k(Z_3, Y_n)} \tag{4.18}$$

$$V_{2r} = V_g \left[\frac{k(Z_{2r+1}, Y_n)}{k(Z_3, Y_n)}\right] \tag{4.19}$$

$$I_3 = V_g \left[\frac{k(Y_4, Y_n)}{k(Z_3, Y_n)}\right] \tag{4.20}$$

and

$$I_{2r+1} = I_3 \left[\frac{k(Y_{2r+2}, Y_n)}{k(Y_4, Y_n)}\right] \tag{4.21}$$

5. First and Second Order RC Ladder Networks.

It is of interest to a design engineer to be able to synthesize an electric network from a given voltage-ratio transfer function. The problem is that of determining the values of circuit components such that they will provide the desired response when connected together to form a circuit. A method for determining these values for a first Cauer form of RC ladder network, such as shown in Fig. 2., will now be illustrated by starting with a single resistor-capacitor network and then progressively adding on more components.

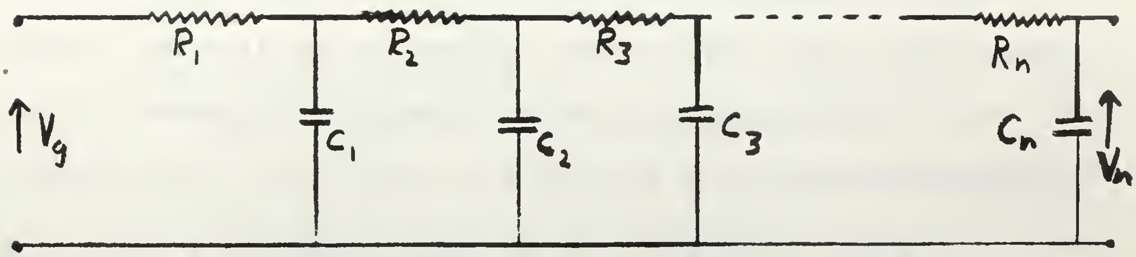


Fig. 2. RC Ladder Network of the First Cauer Form

The voltage-ratio transfer function of the network as given by Eq. (4.16) as a function of the complex frequency variable, s, is

$$\frac{V_n}{V_g} = \frac{1}{Q_m(s)} = \frac{1}{k(R_1, sC_n)} \tag{5.1}$$

For a two-component network the denominator polynomial is

$$Q_2(s) = R_1C_1s+1 = T_{11}s+1 \tag{5.2}$$

Define a L-section time constant, T_{ii}, as

$$T_{ii} = R_iC_i \tag{5.3}$$

for various values of the subscripted variables. If we are given a transfer function whose denominator polynomial is of the form

$$Q_2(s) = K_1s+1 \tag{5.4}$$

where K_1 is some positive real constant, and we wish to synthesize a network for this function, then we can set

$$R_1C_1 = K_1$$

and get

$$R_1 = \frac{K_1}{C_1} \tag{5.5}$$

By assigning a positive value to C_1, we can calculate a value for R_1, and hence will have the necessary component values to synthesize the network.

Thus far this has been a rather trivial problem. Adding on another resistor and capacitor will complicate the situation somewhat.

For a four-component network the denominator polynomial of the transfer function is

$$Q_4(s) \quad = \quad T_{11}T_{22}s^2 \quad + \quad (T_{11} + T_{12} + T_{22})s + 1 \tag{5.6}$$

The denominator polynomial of a given transfer function would be of the form

$$Q_4(s) \quad = \quad K_1 s^2 \quad + \quad K_2 s \quad + 1 \tag{5.7}$$

where

$$K_1 \quad = \quad T_{11}T_{22}$$

$$K_2 \quad = \quad T_{11} \quad + \quad T_{12} \quad \pm \quad T_{22} \tag{5.8}$$

From Eqs. (5.8) we get

$$T_{12} \quad = \quad K_2 \quad - \quad T_{11} \quad - \quad T_{22} \tag{5.9}$$

Let

$$T_{11} \quad = \quad a \quad = \quad R_1C_1$$

where <u>a</u> is some positive real constant. (Note: negative values of <u>a</u> have no meaning since this would imply a negative time constant in the ladder network). Eq. (5.9) can be rewritten as

$$T_{12} \quad = \quad K_2 - a - \frac{K_1}{a} \quad = \quad R_1C_2 \tag{5.10}$$

Define T_{ij} as a cross time constant. It is the product of R_i and C_j for all values of the subscripted variables except for i equal to j. All time constants for the network must be positive since we consider only positive values for the R's and C's. The requirement that $T_{12} > 0$ will be satisfied if the values of <u>a</u> are restricted to those positive values for which

$$K_2 \quad > a + \frac{K_1}{a} \tag{5.11}$$

We can define a function, f(a), from Ineq. (5.11) as

$$f(a) = K_2 - a - \frac{K_1}{a} > 0 \tag{5.12}$$

A typical plot of the function is shown in Fig. 3. for assumed values of K_1 equal to ten and K_2 equal to twenty. Since we have stated that f(a) must be greater than zero, then the graph can be used to determine those values of a for which the restriction on f(a) is satisfied.

Using Eqs. (5.8) and (5.10) and the time constant definition of T_{11}, we can write the following:

$$\begin{aligned} C_2 &= \frac{K_2 - a - \frac{K_1}{a}}{R_1} \\ C_1 &= \frac{a}{R_1} \\ R_2 &= \frac{K_1}{aC_2} \end{aligned} \tag{5.13}$$

By assigning a positive value to R_1 and using Eqs. (5.13), the values of C_1, C_2, and R_2 can be calculated to synthesize the desired network.

To illustrate with an example, assume that we are given a voltage-ratio transfer function whose denominator polynomial is

$$Q_4(s) = 10s^2 + 20s + 1$$

where

$$K_1 = 10$$

$$K_2 = 20$$

Fig. 3. was plotted for these values of K_1 and K_2, therefore it can be used to determine an allowable value for the constant, a. We see from Fig. 3, that all values of a from about 0.5 to 19.5 result in f(a) being greater than zero, hence these are all allowable values. If we choose a to be unity and assign a value of 9 megohms to R_1, then Eqs. (5.13) will give

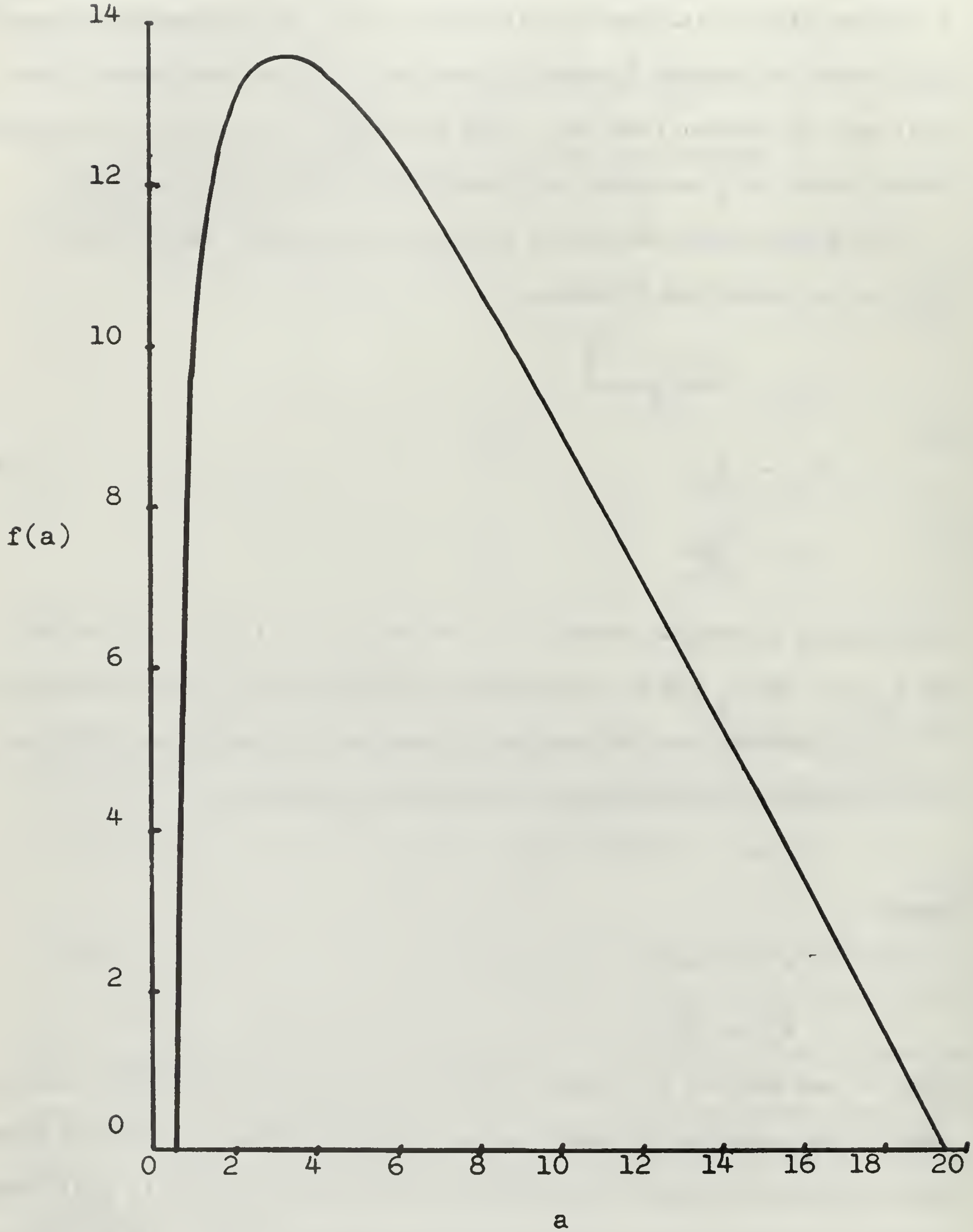


Fig. 3. Range of Allowable Values of Time Constant a.

$$C_2 = \frac{20 - 1 - 10}{9 \times 10^6} = 10^{-6} \text{ farads}$$

$$R_2 = \frac{10}{10^{-6}} = 10 \text{ megohms}$$

$$C_1 = \frac{1}{9 \times 10^6} = \frac{1}{9} \times 10^{-6} \text{ farads}$$

The component values can be substituted into Eqs. (5.8) to insure that they do in fact give the desired transfer function.

6. Third Order RC Ladder Networks.

Synthesizing a two-component ladder network was seen to be a trivial problem. Adding two more components to the network made the problem a little more complex, but still quite straight-forward. Adding a third resistor and capacitor to form a six-component network will now be discussed at length to better illustrate the synthesis procedure.

For a six-component network the denominator polynomial of a given voltage-ratio transfer function is

$$Q_6(s) = K_1 s^3 + K_2 s^2 + K_3 s + 1 \tag{6.1}$$

where

$$\begin{aligned} K_1 &= T_{11}T_{22}T_{33} \\ K_2 &= T_{11}T_{22} + T_{11}T_{23} + T_{11}T_{33} + T_{12}T_{33} + T_{22}T_{33} \\ K_3 &= T_{11} + T_{12} + T_{13} + T_{22} + T_{23} + T_{33} \end{aligned} \tag{6.2}$$

Let

$$T_{11} = a \quad \text{and} \quad T_{33} = b$$

where a and b are some positive real constants. Eqs. (6.2) can be rewritten as

$$K_1 = ab\, T_{22} \tag{6.3a}$$

$$K_2 = aT_{22} + aT_{23} + ab + bT_{12} + bT_{22} \tag{6.3b}$$

$$K_3 = a + T_{12} + T_{13} + T_{22} + T_{23} + b \tag{6.3c}$$

Solving for

$$T_{22} = \frac{K_1}{ab} \tag{6.3d}$$

from the first of Eqs. (6.3) and substituting this into the second two gives

$$bT_{12} + aT_{23} = K_2 - ab - \frac{K_1}{b} - \frac{K_1}{a} \tag{6.4a}$$

$$T_{12} + T_{13} + T_{23} = K_3 - a - b - \frac{K_1}{ab} \tag{6.4b}$$

If the network is to be physically realizable, then

$$K_2 - ab - \frac{K_1}{b} - \frac{K_1}{a} > 0 \tag{6.5a}$$

$$K_3 - a - b - \frac{K_1}{ab} > 0 \tag{6.5b}$$

<u>must</u> hold since all values of the R's and C's are to be positive. The left side of Eqs. (6-4) are therefore greater than zero. Ineqs. (6.5) can be considered as necessary conditions on allowable values for <u>a</u> and <u>b</u>. It should be noted that these inequalities <u>will</u> hold if the given transfer function was derived from an <u>actual</u> RC ladder network of the first Cauer form. (This can be proven by using simple inequalities and the fact that we started out with all positive values in Eqs. (6.3)).

Investigation of Ineqs. (6.5) should prove to be instructive at this point since they were stated to be necessary conditions for physical realizability of the network. If we set these inequalities equal to zero and solve for <u>a</u> in terms of <u>b</u> for both equations, then we can plot <u>a</u> as a function of <u>b</u> for given values of K_1, K_2, and K_3. The plots will show allowable values for <u>a</u> and <u>b</u>.

Setting Ineqs. (6.5) equal to zero and solving for a in terms of b gives

$$a = \frac{(K_1 - K_2 b) \pm \sqrt{(K_2 b - K_1)^2 - 4K_1 b^3}}{-2b^2} \qquad (6.6a)$$

from Ineq. (6.5a) and

$$a = \frac{(b^2 - K_3 b) \pm \sqrt{(K_3 b - b^2)^2 - 4K_1 b}}{-2b} \qquad (6.6b)$$

from Ineq. (6.5b). If we had solved for b in terms of a, we would have gotten

$$b = \frac{(K_1 - K_2 a) \pm \sqrt{(K_2 a - K_1)^2 - 4K_1 a^3}}{-2a^2} \qquad (6.7a)$$

from Ineq. (6.5a) and

$$b = \frac{(a^2 - K_3 a) \pm \sqrt{(K_3 a - a^2)^2 - 4K_1 a}}{-2a} \qquad (6.7b)$$

from Ineq. (6.5b). Note the symmetry between Eqs. (6.6) and (6.7). Allowable values of a and b must be both positive and real, hence the plots are in the first quadrant. The allowable values are restricted to those values which lie within the region bounded by both of the closed curves. Plots of Eqs. (6.6a) and (6.7a) result in identical curves because of the symmetry of the equations. The same is true of plots of Eqs. (6.6b) and (6.7b).

Fig. 4. shows the curves of Eqs. (6.6) for an assumed given case of

$$Q_6(s) = 3s^3 + 9.5s^2 + 8s + 1 \qquad (6.8)$$

where

$$K_1 = 3 \qquad K_2 = 9.5 \qquad K_3 = 8$$

Eqs. (6.4) serve as a basis in attempting to synthesize the network. Suitable values for the constants a and b must be chosen, or be given. Three interesting cases arise in considering the problem. These cases are:

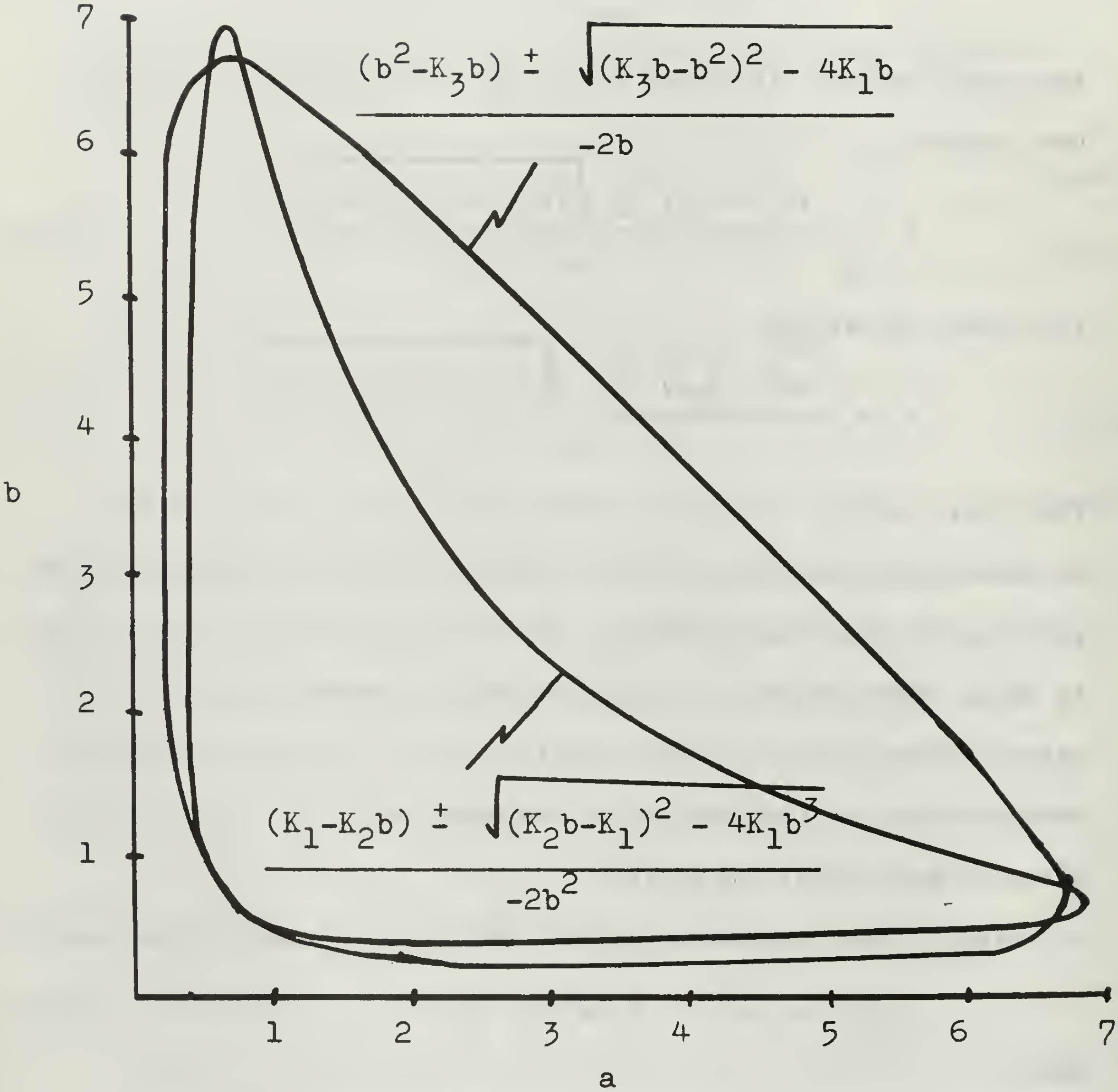


Fig. 4. Range of Allowable Values of Constants a and b

A) We are free to choose values of a and b so long as Ineqs. (6.5) are satisfied.

B) We are given a specific value of a or b and must choose the other such that Ineqs. (6.5) are satisfied.

C) We are given specific values for both a and b.

Each of these three cases will be discussed separately.

A. Constants a and b Not Specified

If a and b are not specified, then we are free to choose them to be any value desired so long as Ineqs. (6.5) are satisfied and the final circuit component values satisfy Eqs. (6.2). At this point it is convenient to let a equal b in order to simplify subsequent equations. (Justification for assuming that a equals b lies in the symmetry of Eqs. (6.6) and (6.7). If some value of a is allowable then certainly that same value of b is also allowable as far as Ineqs. (6.5) are concerned.) Eqs. (6.4) can then be rewritten as

$$T_{12} + T_{23} = \frac{K_2}{a} - a - \frac{2K_1}{a^2} \tag{6.9a}$$

$$T_{12} + T_{13} + T_{23} = K_3 - 2a - \frac{K_1}{a^2} \tag{6.9b}$$

Subtracting the first equation from the second gives

$$T_{13} = K_3 - \frac{K_2}{a} - a + \frac{K_1}{a^2} \tag{6.10}$$

Since T_{13} must be a positive real constant, then

$$K_3 - \frac{K_2}{a} - a + \frac{K_1}{a^2} > 0 \tag{6.11}$$

must also be satisfied. Ineq. (6.11) can be considered as a third necessary condition on the allowable values of a. Those values which satisfy Ineqs. (6.5) must also satisfy Ineq. (6.11) in order to be allowable. A plot of Ineq. (6.11) as a function of a for given values

of K_1, K_2, and K_3 can be made to find the values which are allowable. (As an alternative, values of a which will satisfy Ineq. (6.11) are found on the segment of a line of unit slope through the origin of the plots of Eqs. (6.6) that is bounded by the innermost curves). Any allowable value of a can be used in the synthesis procedure.

After determining a suitable value of a, we can calculate T_{13} from Eq. (6.10) for the given values of K_1, K_2, and K_3. By definition,

$$T_{13} = R_1C_3$$

or

$$C_3 = \frac{T_{13}}{R_1} \tag{6.12}$$

Assigning a positive value to R_1 allows us to calculate C_3. C_1 and R_3 can be found from

$$C_1 = \frac{a}{R_1}$$

and

$$R_3 = \frac{a}{C_3}$$

since a = b and by definition

$$T_{11} = R_1C_1 = a$$

and

$$T_{33} = R_3C_3 = b$$

Note that in effect, Eq. (6.12) gives a relationship between time constants T_{11} and T_{33} by giving a relationship between R_1 and C_3.

We have thus far found suitable values for four of the six circuit components. We must still find values for R_2 and C_2 in order to synthesize the network. These can be determined from Eq. (6.9a) and the fact that

$$T_{22} = R_2C_2 = \frac{K_1}{a^2}$$

In Eq. (6.3d). Substituting

$$C_2 = \frac{K_1}{R_2 a^2} \tag{6.13}$$

into Eq. (6.9a) and using time-constant definitions give

$$\frac{R_1 K_1}{R_2 a^2} + R_2 C_3 = \frac{K_2}{a} - a - \frac{2K_1}{a^2} \tag{6.14}$$

from which we can obtain an expression for R_2 by using the Quadratic Formula. Then

$$R_2 = \frac{Y \pm \sqrt{Y^2 - \frac{4K_1 T_{13}}{a^2}}}{2C_3} \tag{6.15}$$

where Y is defined as

$$Y = \frac{K_2}{a} - a - \frac{2K_1}{a^2} \tag{6.16}$$

as a convenience in writing Eq. (6.15). Knowing values for a, K_1, K_2, R_1, and C_3 allows us to solve for R_2. Finally, we can find C_2 from Eq. (6.13) and our work is finished.

In calculating R_2 using Eq. (6.15), it should be noted that only those values of $\underline{a}$ which were previously determined to be allowable can actually be used in the synthesis procedure for which

$$a^2 Y^2 - 4K_1 R_1 C_3 \geqslant 0 \tag{6.17}$$

Since we cannot have a complex resistance. Ineq. (6.17) represents a fourth restriction on allowable values of $\underline{a}$. It can be plotted as a function of $\underline{a}$ to determine which previously allowable values are still allowable.

We have in effect then been able to take Eqs. (6.2), which are three equations in six unknowns, and solve for the six unknowns. This was done by finding suitable values for two time constants $\underline{a}$ and $\underline{b}$ (where $\underline{a}$ was

assumed to be equal to b), assigning a value to one of the unknowns, R_1, and solving for the other five unknowns in terms of R_1 and a such that Eqs. (6.2) are satisfied.

It might seem at first glance that finding values of a which satisfy the four restrictions given by Ineqs. (6.5), (6.11), and (6.17) would prove to be so laborious as to make the synthesis procedure of little value. Such is not the case, especially if one has access to a digital computer. (Making the necessary plots by hand might require a bit of work, but the task is quite straight-forward.) It should be noted from Eqs. (6.15) and (6.13) that for each allowable value of a and each assigned value of R_1, we get two sets of values of R_2 and C_2 which can be used to synthesize the network. It should be noted that there generally will be any number of allowable values of a for some given transfer function, hence any number of different sets of component values that can be found to do the job.

As an example of the procedure, assume that we have a given transfer function whose denominator polynomial is the same as that given by Eq. (6.8). The plots of Ineqs. (6.5) for

$$K_1 = 3 \qquad K_2 = 9.5 \qquad K_3 = 8$$

shown in Fig. 4. can be used in determining allowable values for a. A convenient choice for a might be unity since this would help reduce the mathematical calculations, and unity is an allowable value as shown by Fig. 4. Using Eq. (6.10), we have

$$T_{13} = K_3 - \frac{K_2}{1} - 1 + \frac{K_1}{1} = (8-9.5-1+3) = 0.5$$

If we let

$$R_1 = 0.5 \text{ ohm}$$

then

$$C_3 = \frac{T_{13}}{R_1} = \frac{0.5}{0.5} = 1 \text{ farad}$$

By using the initial assumption that

$$a = T_{11} = R_1C_1$$

and

$$b = T_{33} = R_3C_3 = a$$

we can determine that

$$C_1 = \frac{a}{R_1} = \frac{1}{0.5} = 2 \text{ farads}$$

and

$$R_3 = \frac{a}{C_3} = \frac{1}{1} = 1 \text{ ohm}$$

Using Eq. (6.16), we find that

$$Y = (9.5 - 1 - 6) = 2.5$$

Eq. (6.15) gives

$$R_2 = \frac{2.5 \pm \sqrt{(2.5)^2 - 4(0.5)3}}{2} = \frac{2.5 \pm 0.5}{2} = 1;\ 1.5 \text{ ohms}$$

For R_2 equal to one ohm,

$$C_2 = \frac{K_1}{R_2a^2} = \frac{3}{1} = 3 \text{ farads}$$

and for R_2 equal to one and one-half ohms,

$$C_2 = \frac{3}{1.5} = 2 \text{ farads}$$

The component values for the synthesized network are then:

Network #1		Network #2
$R_1 = 0.5$ ohm		$R_1 = 0.5$ ohm
$R_2 = 1$ ohm		$R_2 = 1.5$ ohms
$R_3 = 1$ ohm	and	$R_3 = 1$ ohm
$C_1 = 2$ farads		$C_1 = 2$ farads
$C_2 = 3$ farads		$C_2 = 2$ farads
$C_3 = 1$ farad		$C_3 = 1$ farad

As a check for Network #1:

$$K_1 = T_{11}T_{22}T_{33} = 3$$

$$K_2 = T_{11}T_{22} + T_{11}T_{23} + T_{11}T_{33} + T_{12}T_{33} + T_{22}T_{33}$$

$$= 3 + 1 + 1 + 1.5 + 3 = 9.5$$

$$K_3 = T_{11} + T_{12} + T_{13} + T_{22} + T_{23} + T_{33}$$

$$= 1 + 1.5 + 0.5 + 3 + 1 + 1 = 8$$

As a check for Network #2:

$$K_1 = 3$$

$$K_2 = 3 + 1.5 + 1 + 1 + 3 = 9.5$$

$$K_3 = 1 + 1 + 0.5 + 3 + 1.5 + 1 = 8$$

These values of K_1, K_2, and K_3 check with those of the given transfer function, and Eqs. (6.2) are satisfied.

B. Constants a or b Specified

A situation might arise in which one wishes to synthesize a network from a given voltage-ratio transfer function with a particular value of a or b to be used. This is equivalent to saying that we wish the network to have a particular value for the time constant of the first or last L-section of the network. The logical thing to do would

seem to be to let a equal b again and see if this value for a satisfies Ineqs. (6.5), (6.11), and (6.17). If so, then the procedure outlined in part A above can be used to synthesize the network. If not, then the procedure outlined in part C to follow can be used.

C. Constants a and b Specified

If we are required to use particular values of a and/or b, or if we did not wish to assume that a equals b, then a more general approach must be used to synthesize the network. We must fall back on Eqs. (6.4) as a basis for developing a procedure. If the specified values of a and b do not satisfy Ineqs. (6.5), we cannot physically realize the network. In addition, if either a or b is specified and we cannot find a value for the other such that Ineqs. (6.5) are satisfied, then again we cannot physically realize the network.

Assume, however, that the values of a and b, either specified or chosen, do satisfy Ineqs. (6.5). The problem now is to synthesize the network using these values.

It is convenient at this point to define

$$W = K_2 - ab - \frac{K_1}{b} - \frac{K_1}{a}$$

and

$$Z = K_3 - a - b - \frac{K_1}{ab}$$

where W and Z are positive real constants if Ineqs. (6.5) are satisfied. Eqs. (6.4) can be rewritten in more compact form as

$$bT_{12} + aT_{23} = W \qquad (6.18a)$$

$$T_{12} + T_{13} + T_{23} = Z \qquad (6.18b)$$

The problem now is to first solve Eqs. (6.3a) and (6.18) for suitable values of R_1, C_2, R_2, and C_3. The procedure will be to solve for three unknowns in terms of the fourth, and then assign a suitable value to that fourth unknown. After determining values for R_1 and C_3, we can readily obtain values for C_1 and R_3 from the time constant definitions

$$T_{11} = R_1C_1 = a$$

and

$$T_{33} = R_3C_3 = b$$

(It should be noted that C_1 and R_3 do not appear in the basic Eqs. (6.3a) and (6.18). The only constraint imposed on the values of C_1 and R_3 is that the product of R_1 and C_1 must equal the allowable value of $\underline{a}$ and the product of R_3 and C_3 must equal the allowable value of $\underline{b}$).

Eqs. (6.3a) and (6.18) can be rewritten as

$$C_2 = \frac{K_1}{abR_2} \tag{6.19a}$$

$$R_1C_2 + \frac{a}{b} R_2C_3 = \frac{W}{a} \tag{6.19b}$$

$$R_1C_1 + R_1C_3 + R_2C_3 = Z \tag{6.19c}$$

Substituting the expression for C_2 from the first equation into the last two gives

$$\frac{R_1K_1}{abR_2} + \frac{a}{b} R_2C_3 = \frac{W}{a} \tag{6.20a}$$

$$\frac{R_1K_1}{abR_2} + R_1C_3 + R_2C_3 = Z \tag{6.20b}$$

Eqs. (6.20) are two equations in three unknowns. Solving Eq. (6.20a) for

$$C_3 = \frac{\dfrac{W}{a} - \dfrac{R_1K_1}{abR_2}}{\dfrac{a}{b} R_2} \tag{6.21}$$

and substituting into Eq. (6.20b) gives

$$\frac{R_1K_1}{abR_2} + R_1 \left[\frac{\dfrac{W}{a} - \dfrac{R_1K_1}{abR_2}}{\dfrac{a}{b} R_2} \right] + R_2 \left[\frac{\dfrac{W}{a} - \dfrac{R_1K_1}{abR_2}}{\dfrac{a}{b} R_2} \right] = Z \tag{6.22}$$

which is one equation in two unknowns. Applying the Quadratic Formula to Eq. (6.22) results in

$$R_2 = R_1 \left[\frac{(abK_1 + Wb^3 - b^2K_1)}{2(a^2b^2Z - Wb^3)} \pm \frac{\sqrt{(abK_1 + Wb^3 - b^2K_1)^2 + 4b^2K_1(Wb^3 - a^2b^2Z)}}{2(a^2b^2Z - Wb^3)} \right] \tag{6.23}$$

The quantity in brackets in Eq. (6.23) can be calculated from the known constants a, b, K_1, K_2, and K_3. It should be noted however that only the allowable values of a and b derived from the plots of Ineqs. (6.5) which result in the quantity under the radical sign being equal to or greater than zero can be used. Assigning a suitable value to R_1 allows us to calculate R_2. Then determine C_3 from Eq. (6.21). The last three component values can be obtained from

$$C_1 = \frac{a}{R_1}$$

$$R_3 = \frac{b}{C_3}$$

and

$$C_2 = \frac{K_1}{abR_2}$$

This then gives values for the six components of the network. Note that again we get two values for R_2 from Eq. (6.23) and hence two values of C_2 from Eq. (6.19a) for each set of allowable values of a and b for every assigned value of R_1.

A slightly different approach was used for finding R_2 in the last case as compared to that used for the case where a = b. However, the results are the same. This can be seen by substituting a = b into Eq. (6.23) and using the definitions of W, Y, and Z and Eq. (6.12). The approach used in the last case is more general, as we shall subsequently see.

7. Fourth Order RC Ladder Networks.

The ideas expressed and techniques developed in synthesizing first, second, and third order RC ladder networks can be extended to higher order systems. This can be illustrated by adding another RC L-section to form a fourth order network. The first thing noticed is that the algebraic expressions are getting longer and more involved. Nevertheless, we can proceed in the same manner as before.

For an eight-component network the denominator polynomial of a given voltage-ratio transfer function is

$$Q_8(s) = K_1 s^4 + K_2 s^3 + K_3 s^2 + K_4 s + 1 \tag{7.1}$$

where

$$K_1 = T_{11}T_{22}T_{33}T_{44} \tag{7.2a}$$

$$K_2 = T_{11}T_{22}T_{33} + T_{11}T_{22}T_{34} + T_{11}T_{22}T_{44} + T_{11}T_{23}T_{44}$$
$$+ T_{11}T_{33}T_{44} + T_{12}T_{33}T_{44} + T_{22}T_{33}T_{44} \tag{7.2b}$$

$$K_3 = T_{11}T_{22} + T_{11}T_{23} + T_{11}T_{24} + T_{11}T_{33} + T_{11}T_{34}$$

$$+ T_{11}T_{44} + T_{12}T_{33} + T_{12}T_{34} + T_{12}T_{44} + T_{13}T_{44} \qquad (7.2c)$$

$$+ T_{22}T_{33} + T_{22}T_{34} + T_{22}T_{44} + T_{23}T_{44} + T_{33}T_{44}$$

$$K_4 = T_{11} + T_{12} + T_{13} + T_{14} + T_{22} + T_{23} + T_{24}$$

$$+ T_{33} + T_{34} + T_{44} \qquad (7.2d)$$

Let

$$T_{11} = a \qquad T_{44} = a \qquad T_{22} = b$$

Setting T_{11} and T_{44} equal to the same constant is justified in that the only restriction on values of C_1 and R_4 is that the products R_1C_1 and R_4C_4 be equal to a. The components C_1 and R_4 do not show up in any of the cross time constants, T_{ij}. After finding proper values for R_1 and C_4, we can always adjust C_1 and R_4 so that their respective products do equal a.

With the L-section time constants thus defined, we can rewrite Eqs. (7.2) as

$$T_{33} = \frac{K_1}{a^2b} \qquad (7.3a)$$

$$\frac{K_1}{ab} T_{12} + a^2T_{23} + abT_{34} = X \qquad (7.3b)$$

$$\left(\frac{K_1}{a^2b} + a\right)T_{12} + aT_{13} + 2aT_{23} + aT_{24} + (a+b)T_{34} + T_{12}T_{34} = Y \qquad (7.3c)$$

$$T_{12} + T_{13} + T_{14} + T_{23} + T_{24} + T_{34} = Z \qquad (7.3d)$$

where

$$X = K_2 - \frac{2K_1}{a} - a^2b - \frac{K_1}{b} > 0$$

$$Y = K_3 - 2ab - a^2 - \frac{2K_1}{ab} - \frac{K_1}{a^2} > 0 \tag{7.4}$$

$$Z = K_4 - 2a - b - \frac{K_1}{a^2 b} > 0$$

The requirement that X, Y, and Z all be greater than zero must be satisfied if the network is to be physically realizable. Ineqs. (7.4) can be considered as necessary conditions on the allowable values of a and b for a given transfer function. Again plots of Ineqs. (7.4) can be made to determine allowable values of a and b for use in the synthesis procedure.

Initially we started out with Eqs. (7.2), which are four equations in eight unknowns. By finding suitable values for a and b, we are able to write Eqs. (7.3), which are four equations in six unknowns. The number of unknowns in the equations can be further reduced by noting the following cross time constant definitions:

$$T_{12} = R_1 C_2 = R_1 \left[\frac{b}{R_2}\right]$$

$$T_{13} = R_1 C_3 = R_1 \left[\frac{K_1}{a^2 b R_3}\right]$$

$$T_{23} = R_2 C_3 = R_2 \left[\frac{K_1}{a^2 b R_3}\right]$$

In the above, C_2 and C_3 have been expressed in terms of R_2, R_3, and known constants. The last three of Eqs. (7.3) can be rewritten as

$$\frac{K_1}{a}\left(\frac{R_1}{R_2}\right) + \frac{K_1}{b}\left(\frac{R_2}{R_3}\right) + abR_3C_4 = X \qquad (7.5a)$$

$$\left(\frac{K_1}{a^2} + ab\right)\frac{R_1}{R_2} + \frac{K_1}{ab}\left(\frac{R_1}{R_3}\right) + \frac{2K_1}{ab}\left(\frac{R_2}{R_3}\right) + aR_2C_4 + (a+b)R_3C_4$$

$$+\ b\left(\frac{R_1R_3C_4}{R_2}\right) = Y \qquad (7.5b)$$

$$b\left(\frac{R_1}{R_2}\right) + \frac{K_1}{a^2b}\left(\frac{R_1}{R_3}\right) + \frac{K_1}{a^2b}\left(\frac{R_2}{R_3}\right) + R_2C_4 + R_3C_4 + R_1C_4 = Z \qquad (7.5c)$$

which are three equations in four unknowns. Then proceed to solve for three of these unknowns in terms of the fourth. It should be noted that only one component of each of the four L-sections of the network shows up in Eqs. (7.5) as one of the unknowns.

Solving Eq. (7.5a) for C_4 and substituting it into Eqns. (7.5b) and (7.5c) gives

$$\left[-\frac{K_1b}{a}R_3^2\right]R_1^2 + \left[(a^2b^2+bX-K_1)R_2R_3^2 - K_1R_2^2R_3\right]R_1 + \Big[-\frac{K_1aR_2^4}{b}$$

$$+\ (K_1+aX-\frac{K_1a}{b})R_2^3R_3 + (aX+bX-abY)R_2^2R_3^2 = 0 \qquad (7.6a)$$

$$\left[-\frac{K_1}{a}R_3\right]R_1^2 + \left[-\frac{K_1}{b}R_2^2 + XR_2R_3 + (ab-\frac{K_1}{a})R_3^2\right]R_1$$

$$+\left[-\frac{K_1}{b}R_2^3 + (\frac{K_1}{a}+X-\frac{K_1}{b})R_2^2R_3 + (X-abZ)R_2R_3^2\right] = 0 \qquad (7.6b)$$

which are two equations in three unknowns. Eq. (7.6a) is quadratic in R_1. It can be solved for R_1 by use of the Quadratic Formula. This will give two values of R_1 to be put into Eq. (7.6b), one at a time, to obtain one equation in the two unknowns R_2 and R_3. Admittedly this last equation will be rather complicated. However, we are able to assign a

value to either R_2 or R_3 and solve for the other, graphically if necessary since this is equivalent to changing the impedance level of the network. After finding values for R_2 and R_3, we can go back and find corresponding values for the other six components of the network.

8. Nth Order RC Ladder Networks.

By now it should be evident that a procedural pattern has been evolved for the synthesis procedure. We can find any number of sets of component values which will satisfy a given voltage-ratio transfer function, but these component values will be restricted to certain ranges. The ranges will be determined by the range of allowable values of the L-section time constants. It should also be obvious that a digital computer is needed if we are to synthesize large order networks.

An analysis of what has taken place in the synthesis of RC ladder networks up to the fourth order shows more clearly the procedural pattern, and how the procedure might be extended to the synthesis of an nth order system.

In the first order network there was an initial system equation in two unknowns. This was Eq. (5.2). There were no necessary conditions involved and no allowable values of L-section time constants to be determined. By assigning a value to one unknown in Eq. (5.5), we could solve for the other.

In the second order network there were two initial system equations in four unknowns. These were Eqs. (5.8). We had one necessary condition to satisfy (Ineq. (5.12)) and one L-section time constant for which to determine allowable values. With this time constant we were able to reduce the two initial system equations to one equation in two unknowns. Assigning a value to one of the unknowns allowed us to find values for the other three using Eqs. (5.13).

The third order system had Eqs. (6.2) as three initial system equations in six unknowns. Ineqs. (6.5) were two necessary conditions which had to be satisfied if the network was to be physically realized. It was necessary to determine allowable values for two L-section time constants which would satisfy these conditions. It was also noted that the two time constants could be made equal to each other if desired. The time constants were used to reduce the three initial system equations to Eqs. (6.3d) and (6.4), which are three equations in four unknowns. We then solved for three of these unknowns in terms of the fourth, assigned a value to the fourth, and subsequently went back and found values for all the others.

In the fourth order system, Eqs. (7.2) were four initial system equations in eight unknowns. There were three necessary conditions given by Ineqs. (7.4) to satisfy. It was necessary to determine allowable values for three L-section time constants which would satisfy these inequalities. It was noted that one can always set the first and last L-section time constants equal to each other. The time constants enabled us to reduce the initial system equations to Eqs. (7.3), which are four equations in six unknowns. By redefining some of the cross time constants, we further reduced the system equations to Eqs. (7.5) and Eq. (7.3a), which are four equations in five unknowns. We again solved for four of these unknowns in terms of the fifth, assigned a value to the fifth, and went back to find values for all the others.

The procedural pattern should now be clear for synthesizing an nth order system. We can expect to have n initial system equations in 2n unknowns. There will be (n-1) necessary conditions to satisfy, and (n-1) L-section time constants to be determined which will satisfy these conditions. By using the (n-1) L-section time constants and redefining some

of the cross time constants, we can reduce the initial system equations to n equations in (n+1) unknowns. These must then be solved for n unknowns in terms of the remaining one. Assigning a value to this remaining unknown will allow us to go back and solve for the other (2n-1) unknowns.

It should be noted that in redefining some of the cross time constants to derive the final n equations in (n+1) unknowns, it is necessary to proceed in such a fashion as to have n of the (n+1) unknowns be one component of each of the L-sections and the other unknown to be the other component of the L-section which was not included in the search for allowable time constants. As an example, Eq. (7.3a) has R_3 and C_3 as unknowns since we found allowable values for the first, second and fourth L-section time constants, and Eqs. (7.5) contain R_1, R_2, R_3, and C_4 as unknowns.

Table I summarizes the observations and remarks of this section.

Table I. Characteristics of RC Ladder Networks of the First Cauer Form

Network Order	Basic Criteria Conditions (Ineqs.)	# of R's	# of C's	# of Initial System Equations	# of L-Section Time Constants to be Determined
1	0	1	1	1	0
2	1	2	2	2	1
3	2	3	3	3	2
4	3	4	4	4	3
n	n-1	n	n	n	n-1

9. Computer-Aided Network Design.

Use of a digital computer greatly facilitates the network synthesis problem. Once a program has been written, it can be used over and over again to obtain any number of suitable networks to synthesize a given transfer function. It can provide ranges of component values from which appropriate values can be selected to synthesize the network. The program can be used for any transfer function of the proper form by simply changing input data cards. If the transfer function cannot be physically realized, the program will tell that too. The programs in Appendices II and III illustrate these points.

The programs in Appendices II and III are written in FORTRAN 63 language for use on the CDC 1604 digital computer. They can easily be adapted to FORTRAN 60 and FORTRAN 4 languages with just minor modifications.

The program in Appendix II is entitled SYNTHNET. It is written for the synthesis of a third order RC ladder network using the procedure of section 6 of this paper. Input data is fed to the computer from two data cards placed at the end of the card deck. Input format is F10. Output format is E11. 4. The first data card has K_1, K_2, and K_3 for the given transfer function punched in columns 1-10, 11-20, and 21-30 respectively. Columns 31-40 contain the desired initial value of the L-section time constant, a, columns 41-50 the amount by which a is to be incremented, and columns 51-60 the desired final value of a. The second data card contains the desired values of R_1 punched in each set of ten consecutive columns from column 1 to column 80. A total of eight values of R_1 can be inputted per card. Two sets of component values will be outputted for each value of R_1 for each allowable value of a determined by the program during its running. The corresponding values of a will

also be outputted. Included in the program is a testing procedure to insure that the component values which are calculated actually do synthesize the network. The original K_1, K_2, and K_3 constants are outputted as DK1, DK2, and DK3. The values of the K's as computed from the component values are outputted as DCK1, DCK2, and DCK3. The DK's and DCK's should be the same respectively if the component values are satisfactory.

A computer run was made with the following as input data:

K_1 = 2.25

K_2 = 10.525

K_3 = 10.3

Starting point of $\underline{a}$ = 0.05

Step size of $\underline{a}$ = 0.05

Stopping point of $\underline{a}$ = 10.0

R_1 = 200 Kiloohms

= 250 Kiloohms

= 1 Megohm

= 150 Kiloohms

= 0.5 Ohms

= 100 Megohms

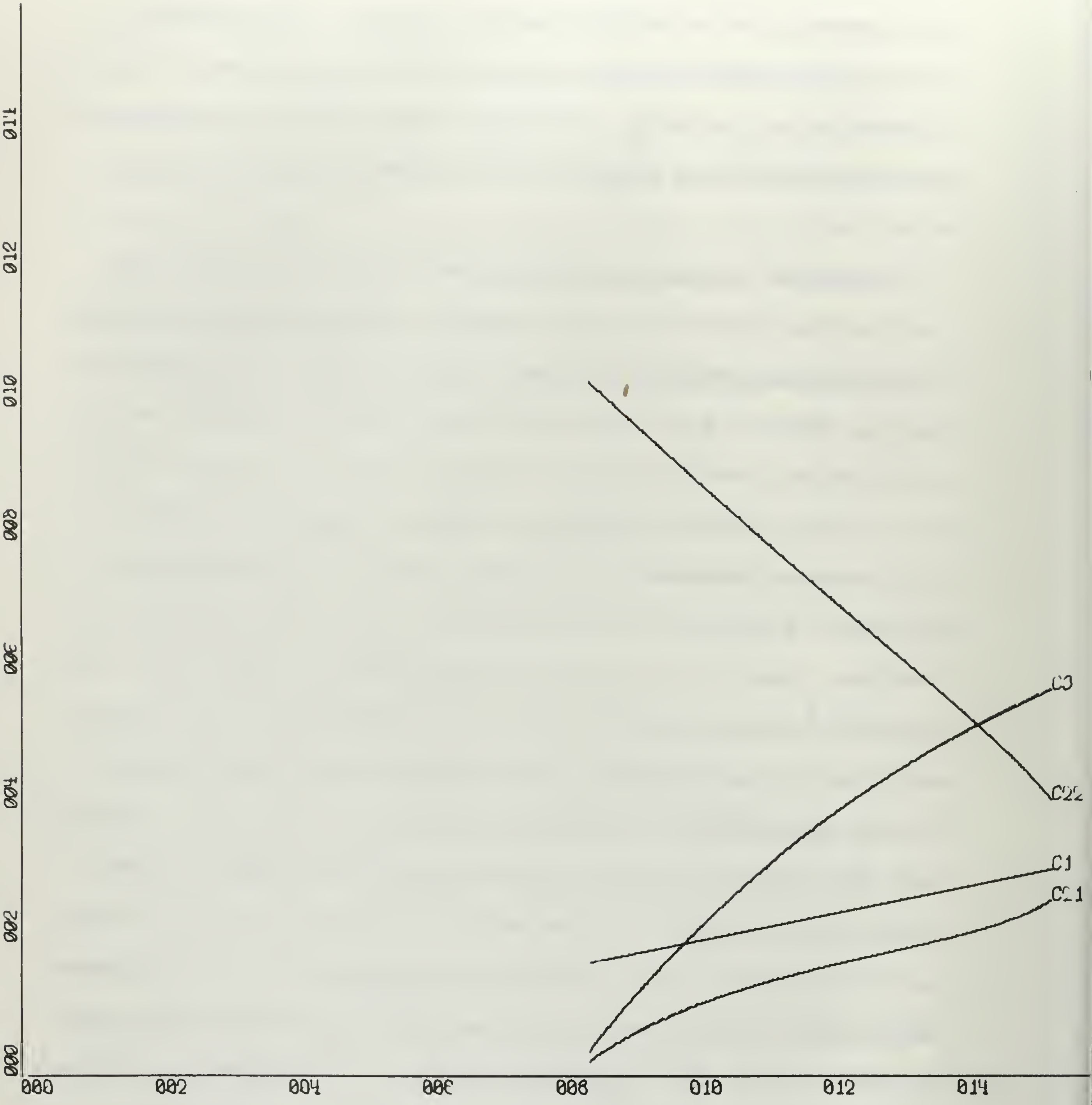
The result was a total of 180 sets of component values, all of which will synthesize the given transfer function.

The program in Appendix III is entitled DRAWCOMP. It is written to do the same task as SYNTHNET, except that its output is in the form of graphs. Input to DRAWCOMP is the same as for SYNTHNET. Each computer run will provide two graphs, one for resistors and one for capacitors. The graphs are plotted with the L-section time constant, $\underline{a}$, along the abscissa and the component values along the ordinate. We

can determine a set of suitable component values for any allowable value of a by extending a line vertically from this a value so as to intersect all of the curves. The proper values for the components are then found at the intersections of the vertical line and the various curves.

A computer run was made with the same input data as before, except that the step size of a was changed to .0004 and the starting and stopping points changed to 0.85 and 1.55 respectively, since these are limiting values of a as shown by the output data of SYNTHNET. Results of the run are shown in Figs. 5.a. through 10.b. It is noted that all of the curves have the same shape, as might be expected. In fact, if we superimpose the curves of Figs. 9.a. and 10.a for the capacitors and Figs. 9.b and 10.b for the resistors it will be seen that they are identical. The only thing different is the ordinate scale factor. Van Valkenburg[1] states in his text that if each resistor in the network is multiplied by some constant, d, and each capacitor by the reciprocal of that same constant, 1/d, then the driving point functions of the network will have been scaled by that factor but the voltage-ratio and current-ratio transfer functions are not affected. This fact is borne out in the graphs. If we input one desired value of R_1 to the computer, we will get a set of curves. If we now input another value of R_1, then in effect we will have scaled the original R_1. The second set of curves will give the same component values as the first set of curves, but they will be scaled. It can be stated then that a single set of curves for a given transfer function will provide a complete set of data for

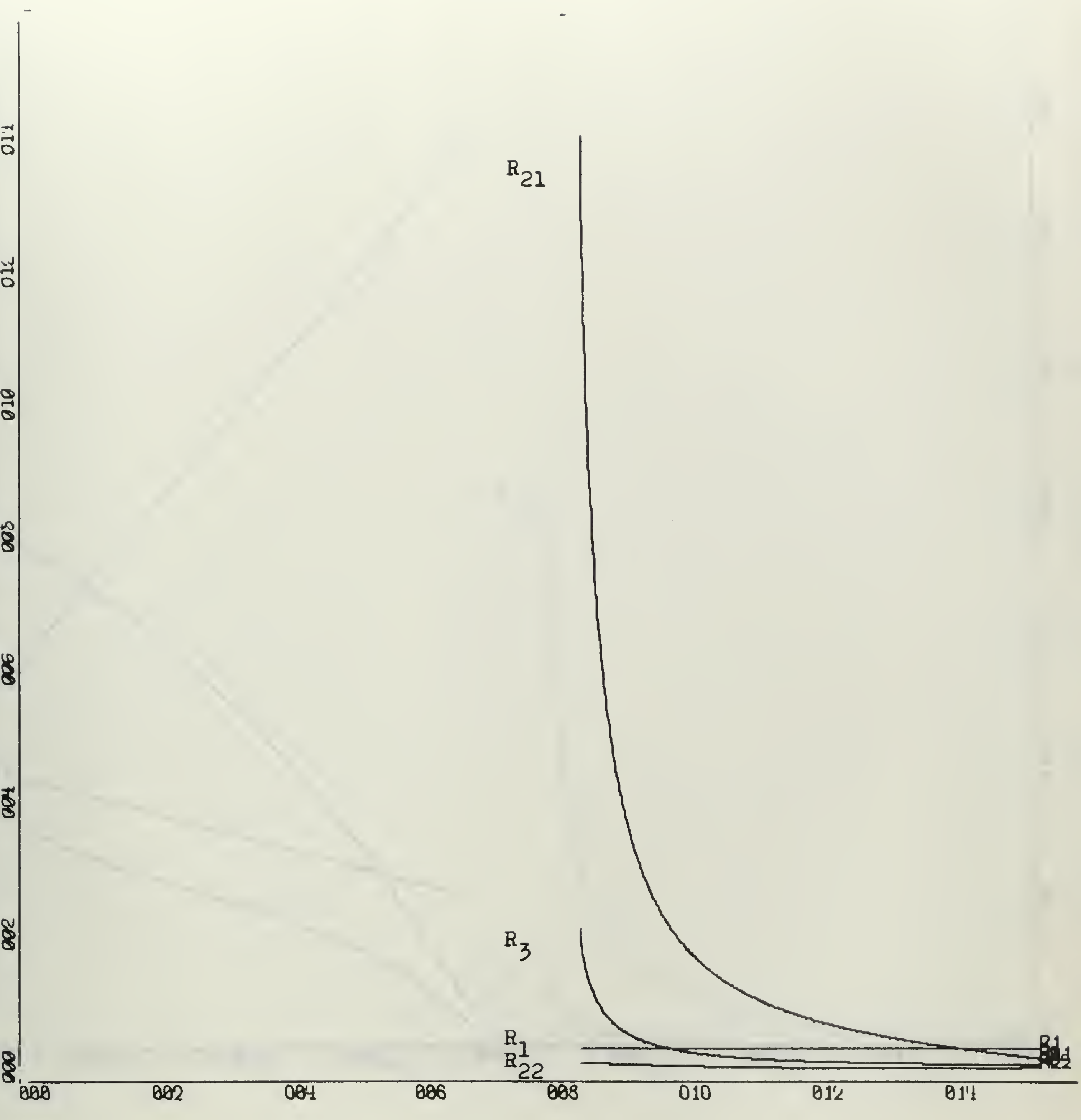
[1]M. E. VanValkenburg, Introduction to Modern Network Synthesis (New York: J. Wiley and Sons, 1960), pp. 48-53.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 2.00E+00 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF C1,C2(1),
C2(2), AND C3 VS TIME CONSTANT, A, FOR CONST. R1

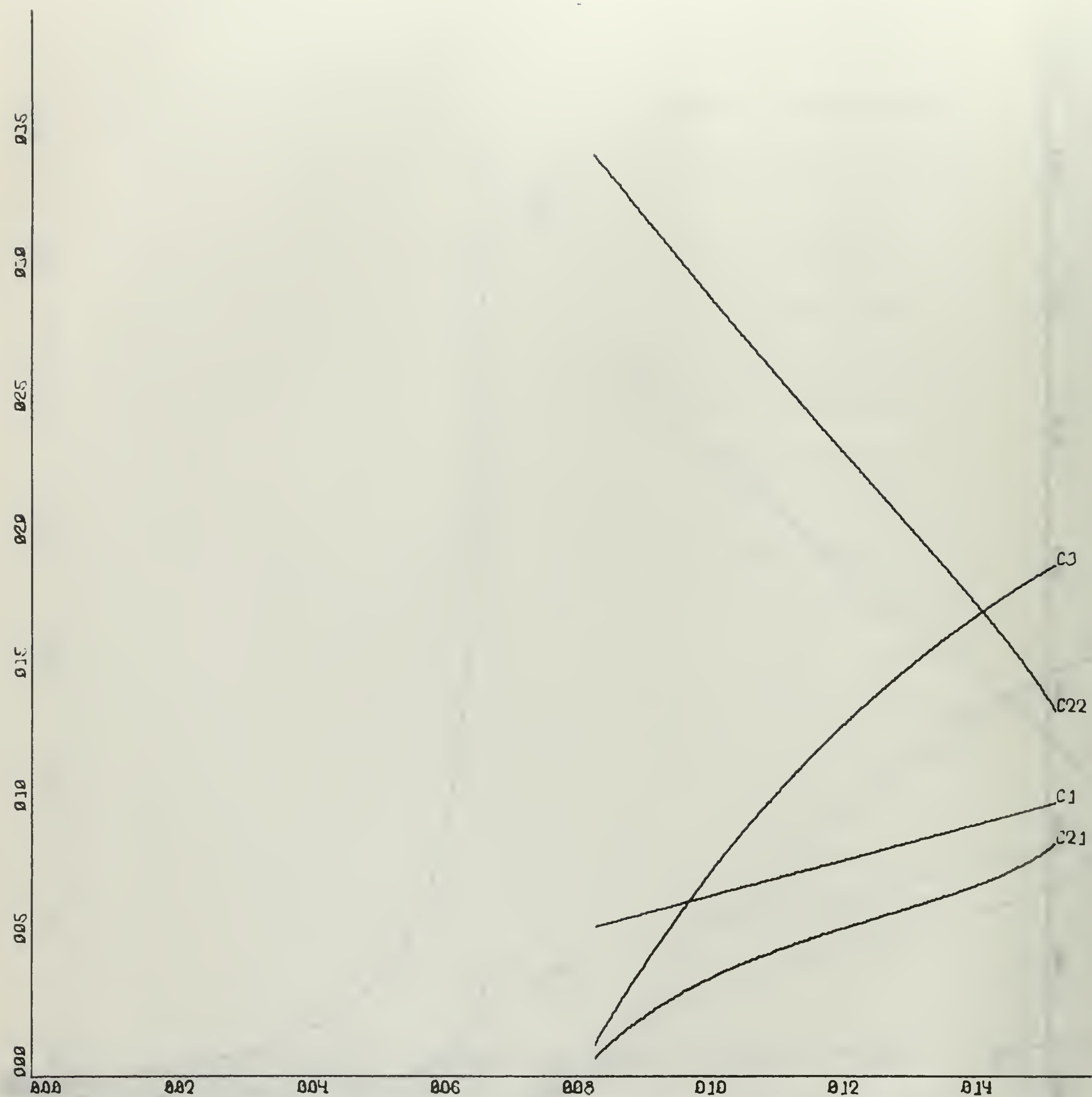
Fig. 5.a. Capacitor Values For R_1 Of 0.5 Ohms.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 2.00E+00 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF R2(1), R2(2), AND R3 VS TIME CONSTANT, A, FOR CONSTANT R1

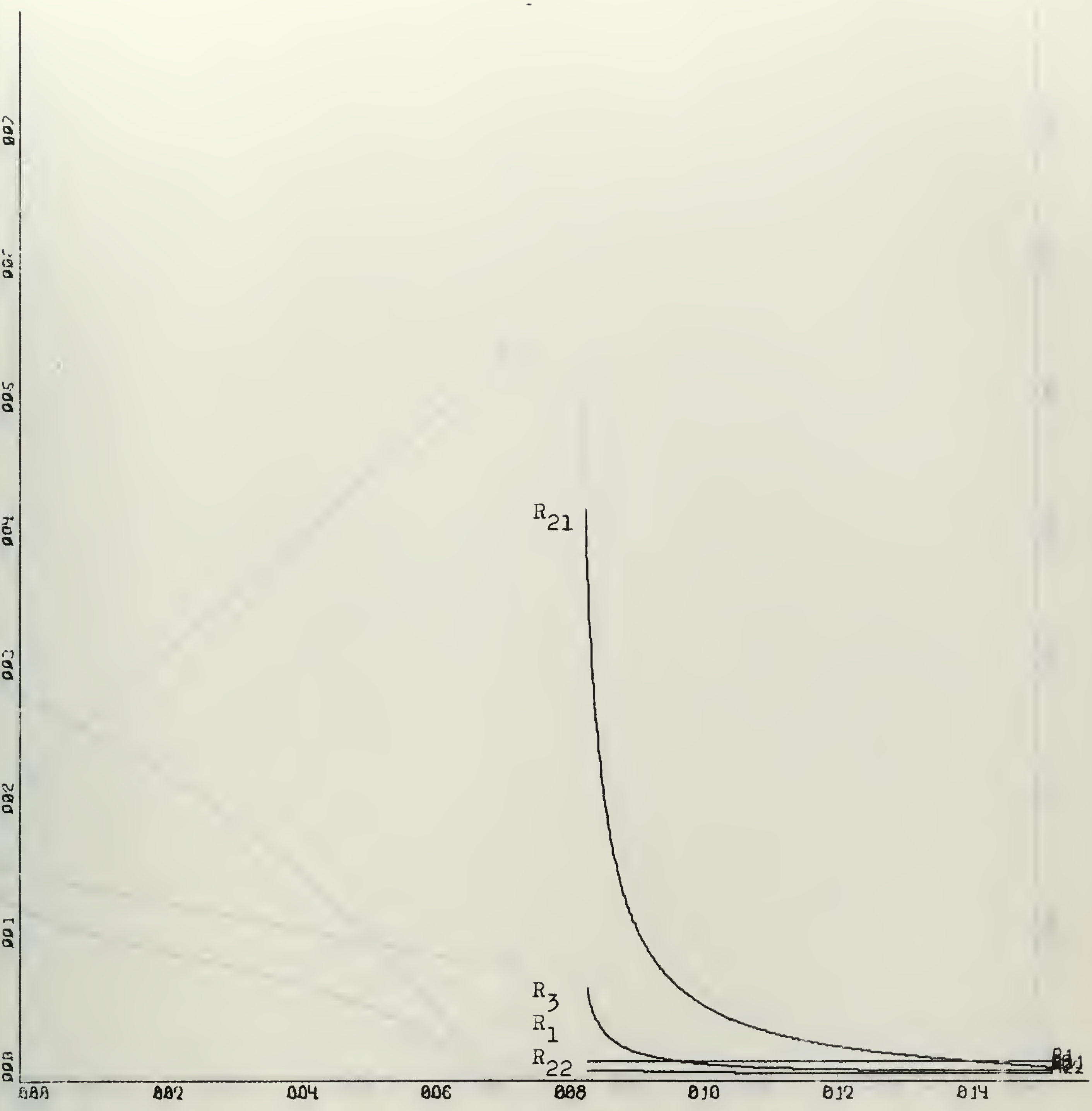
Fig. 5.b. Resistor Values for R_1 of 0.5 Ohms.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 5.00E-06 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF C1, C2(1), C2(2), AND C3 VS TIME CONSTANT, A, FOR CONST. R1

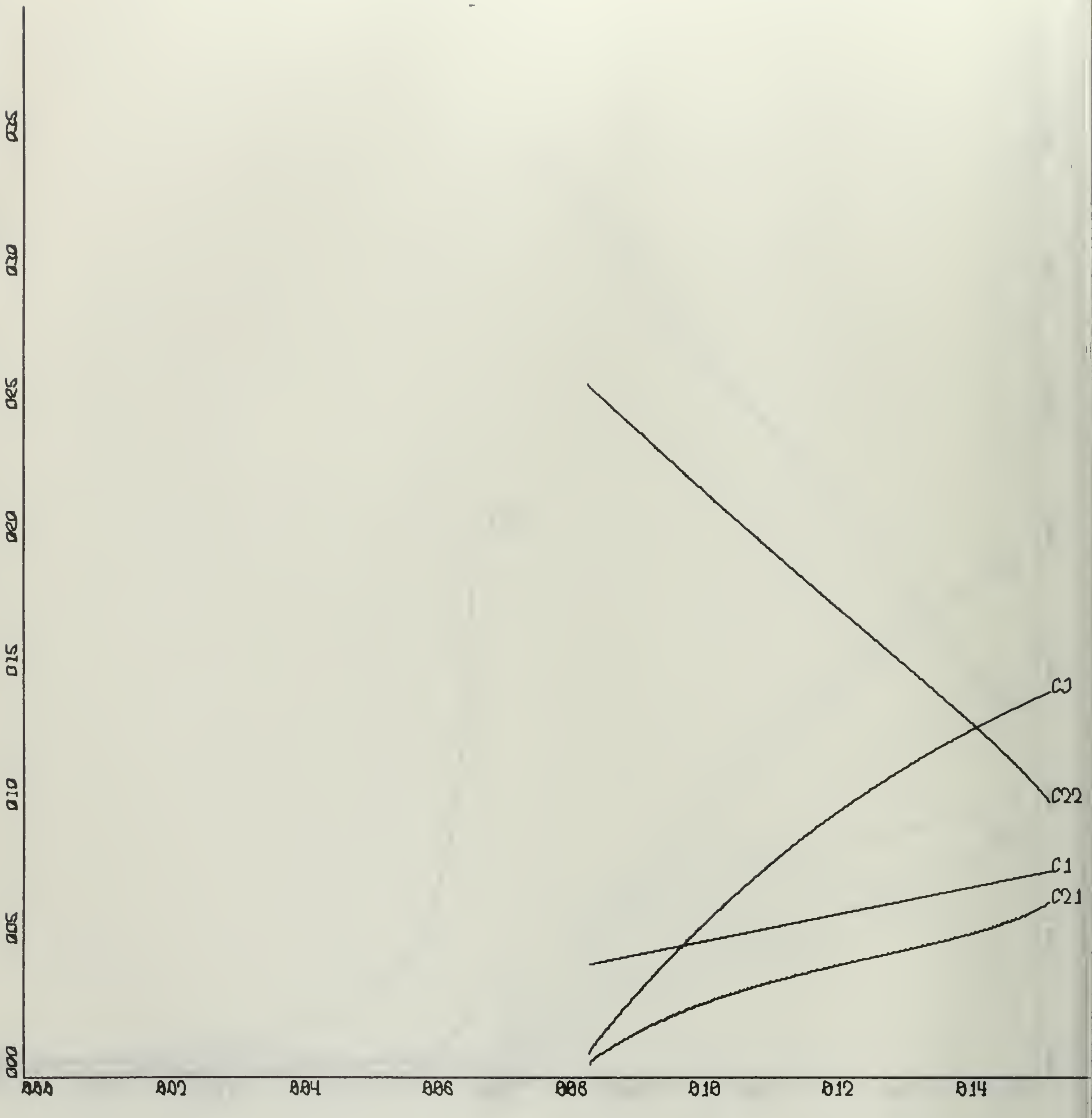
Fig. 6.a. Capacitor Values for R_1 of 150K Ohms.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 1.00E+06 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF R2(1), R2(2), AND R3 VS TIME CONSTANT, A, FOR CONSTANT R1

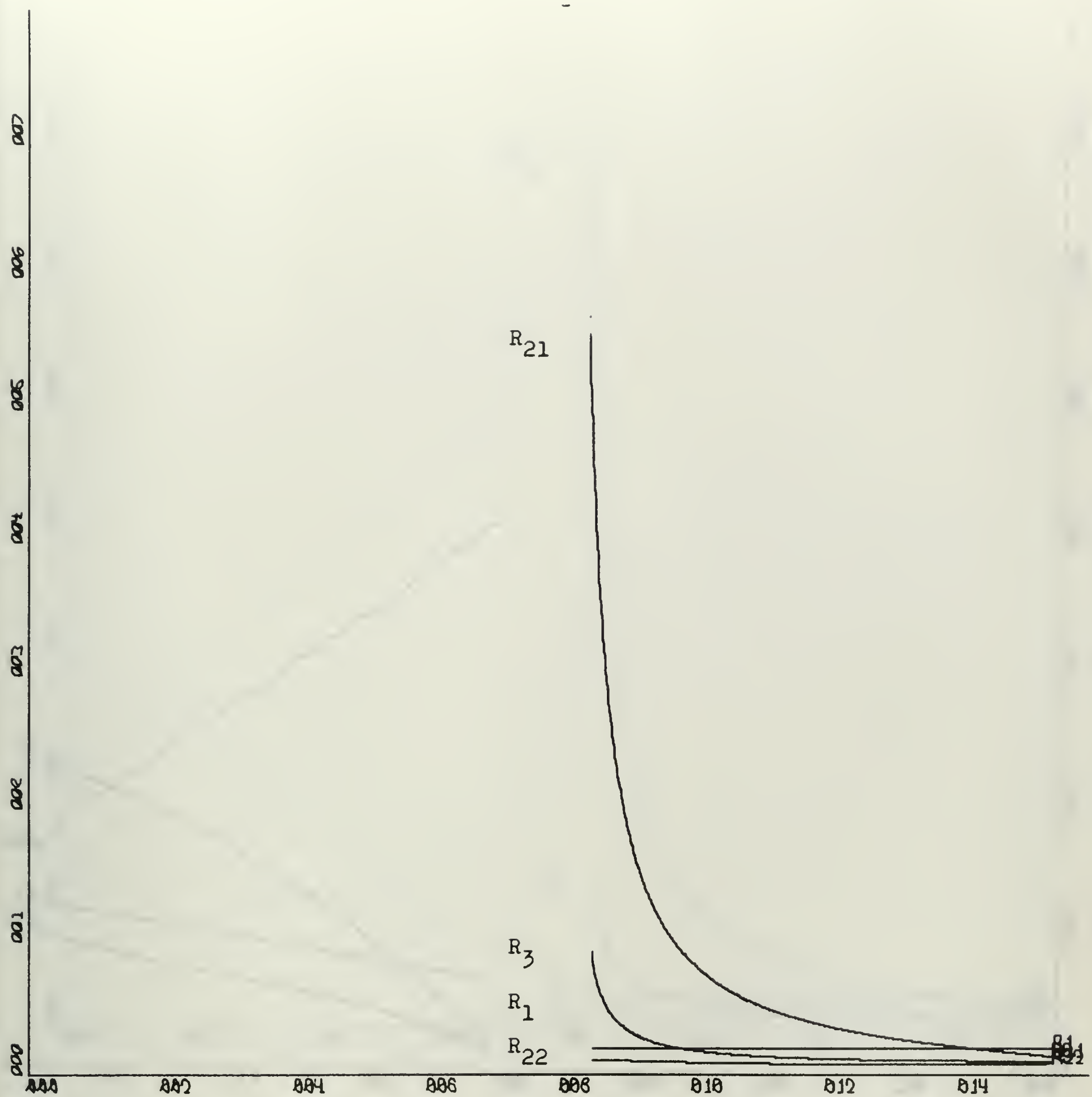
Fig. 6.b. Resistor Values for R_1 of 150K Ohms.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 5.00E-06 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF C1,C2(1), C2(2), AND C3 VS TIME CONSTANT, A, FOR CONST. R1

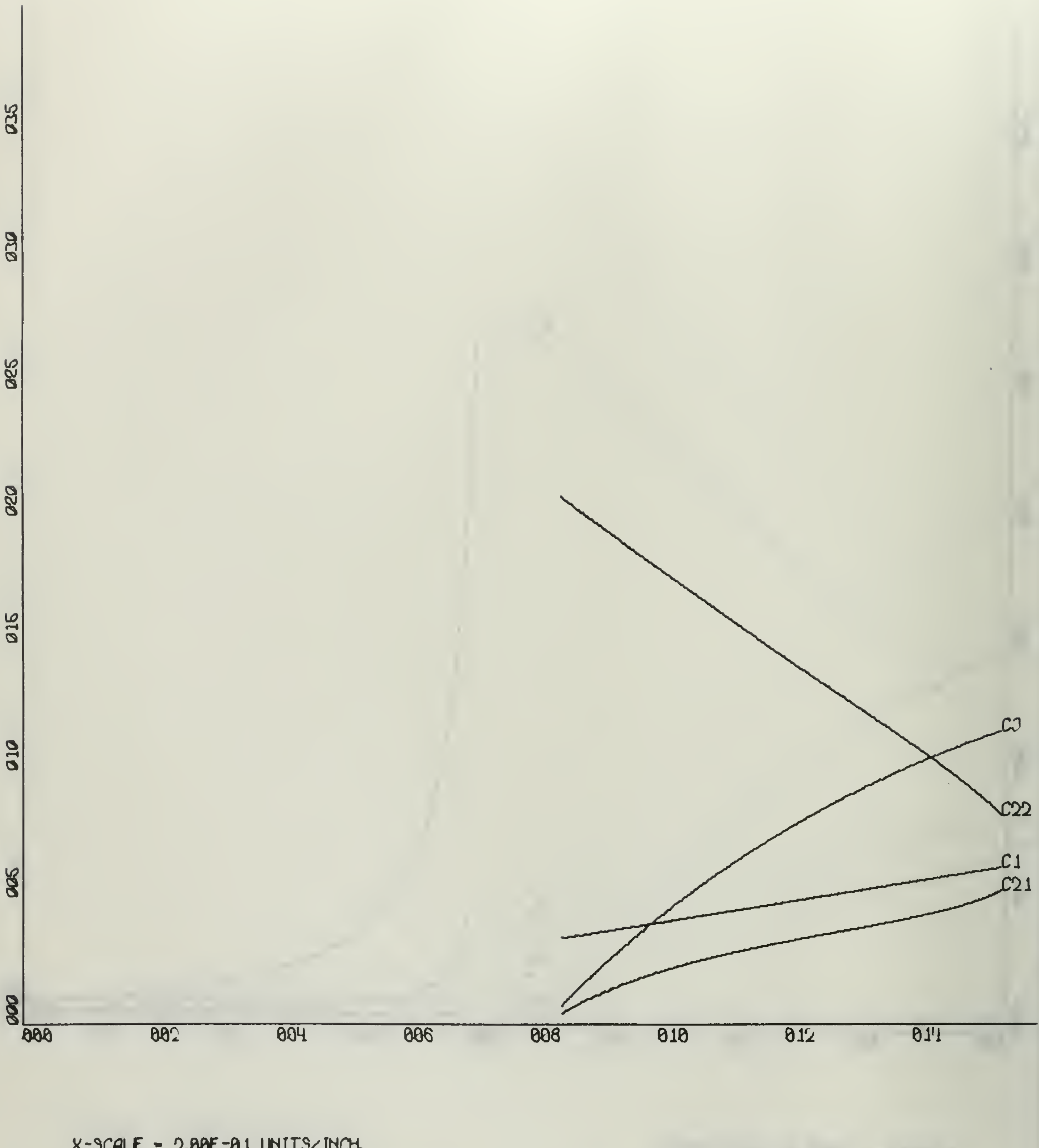
Fig. 7.a. Capacitor Values for R_1 of 200K Ohms.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 1.00E+06 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF R2(1), R2(2), AND R3 VS TIME CONSTANT, A, FOR CONSTANT R1

Fig. 7.b. Resistor Values for R_1 of 200K Ohms.



LILLIS, J. W. THESIS PLOT OF C1,C2(1), C2(2), AND C3 VS TIME CONSTANT, A, FOR CONST. R1

Fig. 8.a. Capacitor Values for R_1 of 250K Ohms.

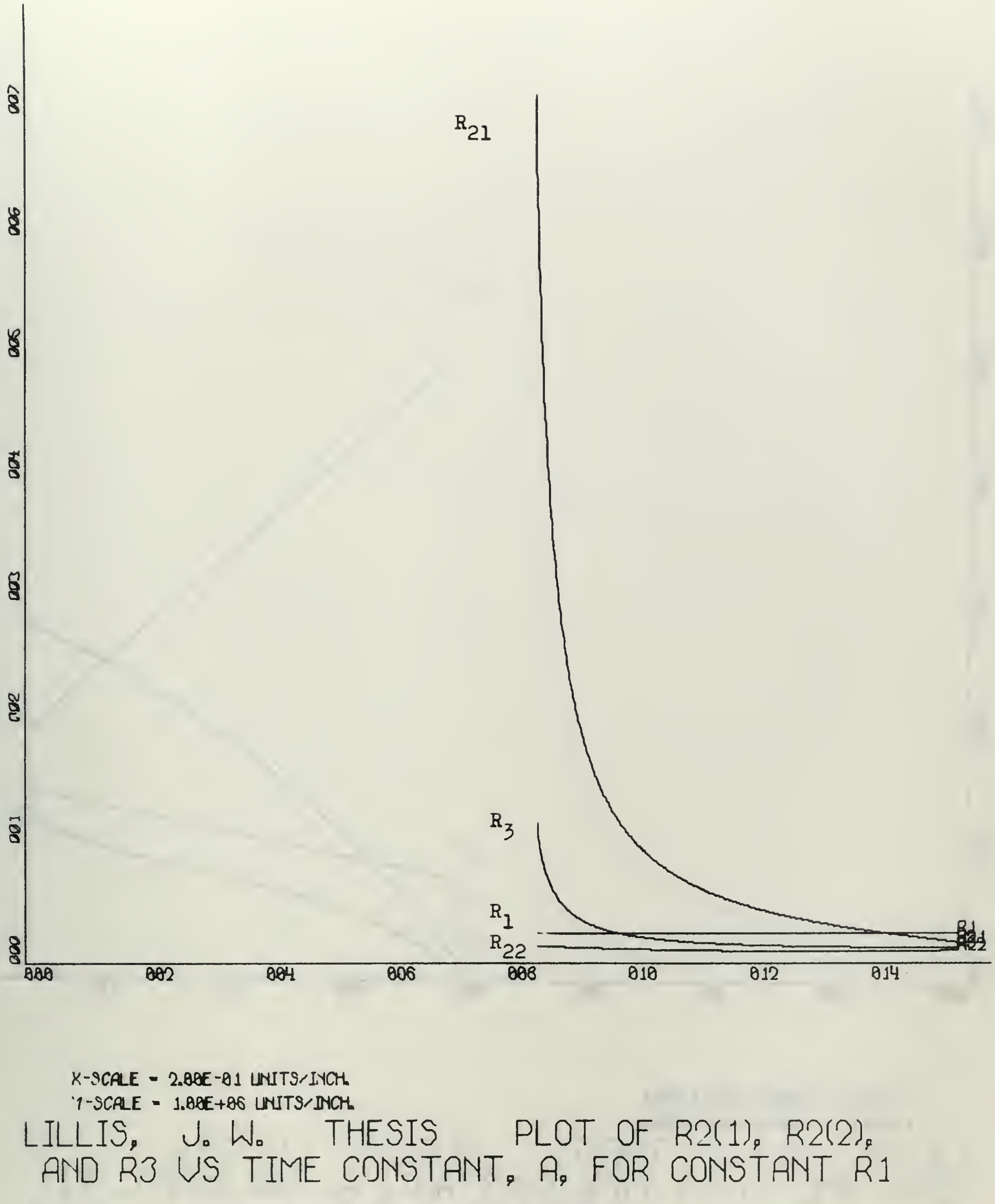
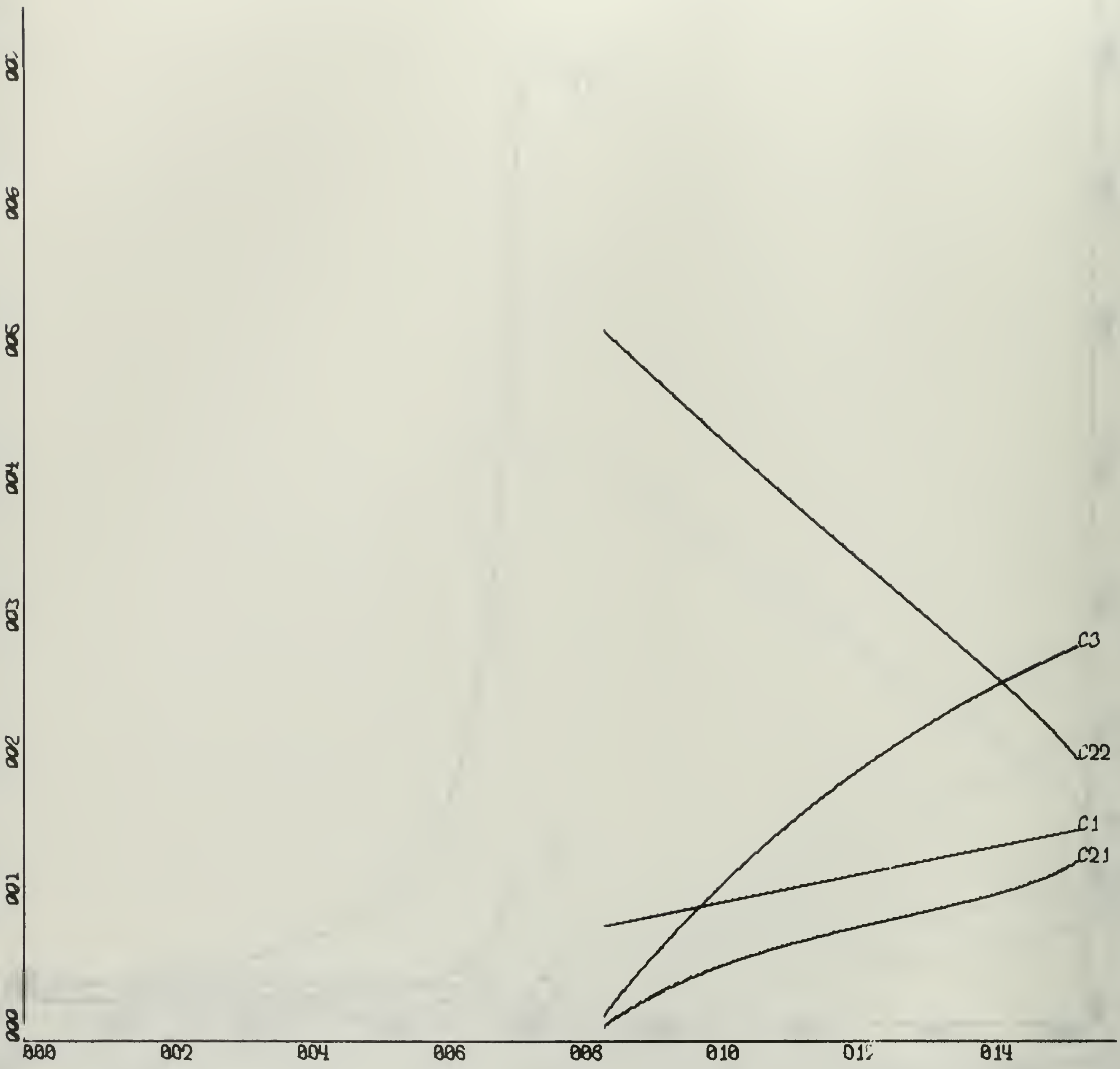


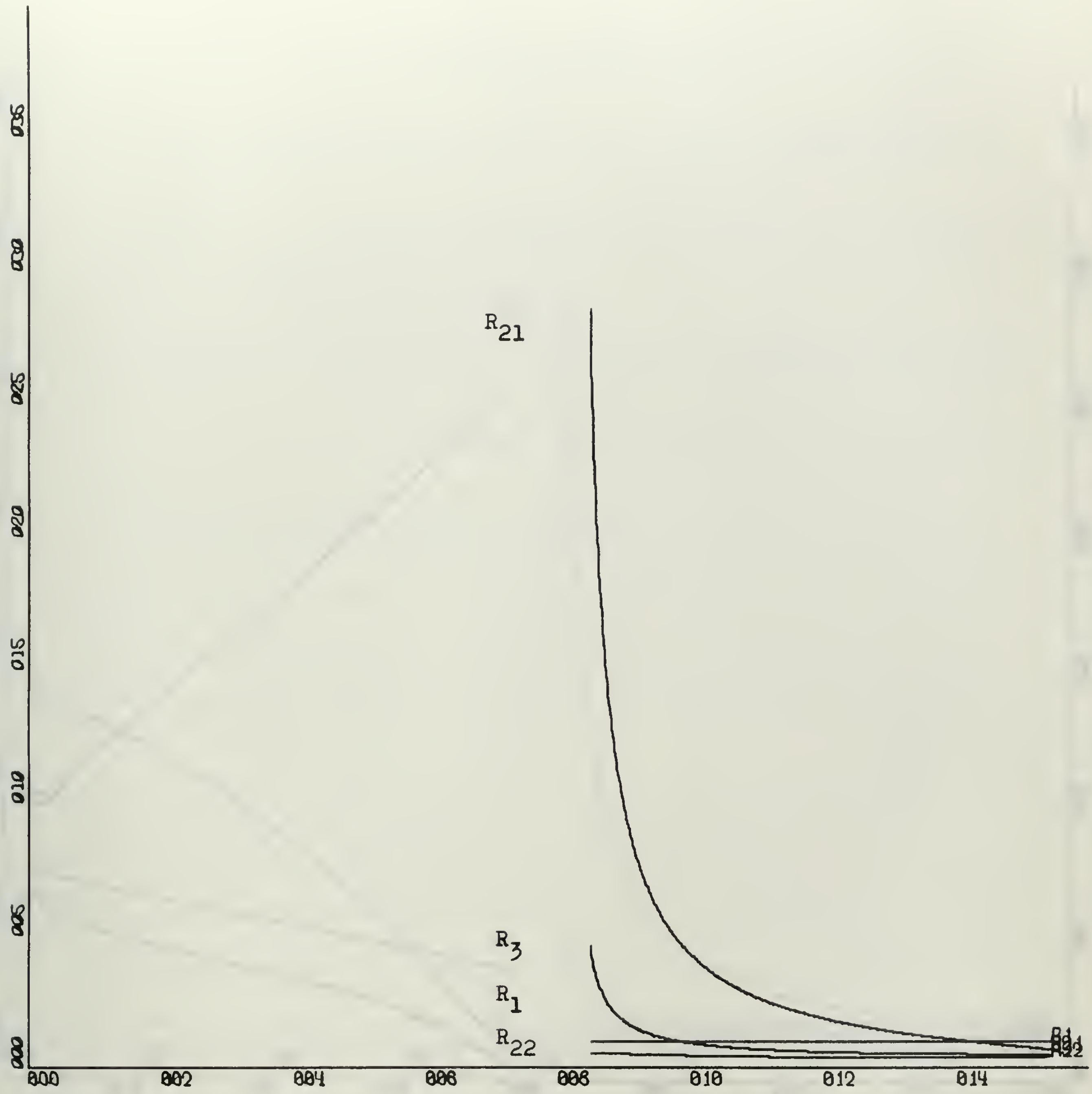
Fig. 8.b. Resistor Values for R_1 of 250K Ohms.



X-SCALE = 2.00E-01 UNITS/INCH
Y-SCALE = 1.00E-06 UNITS/INCH

LILLIS, J. W. THESIS PLOT OF C1,C2(1), C2(2), AND C3 VS TIME CONSTANT, A, FOR CONST. R1

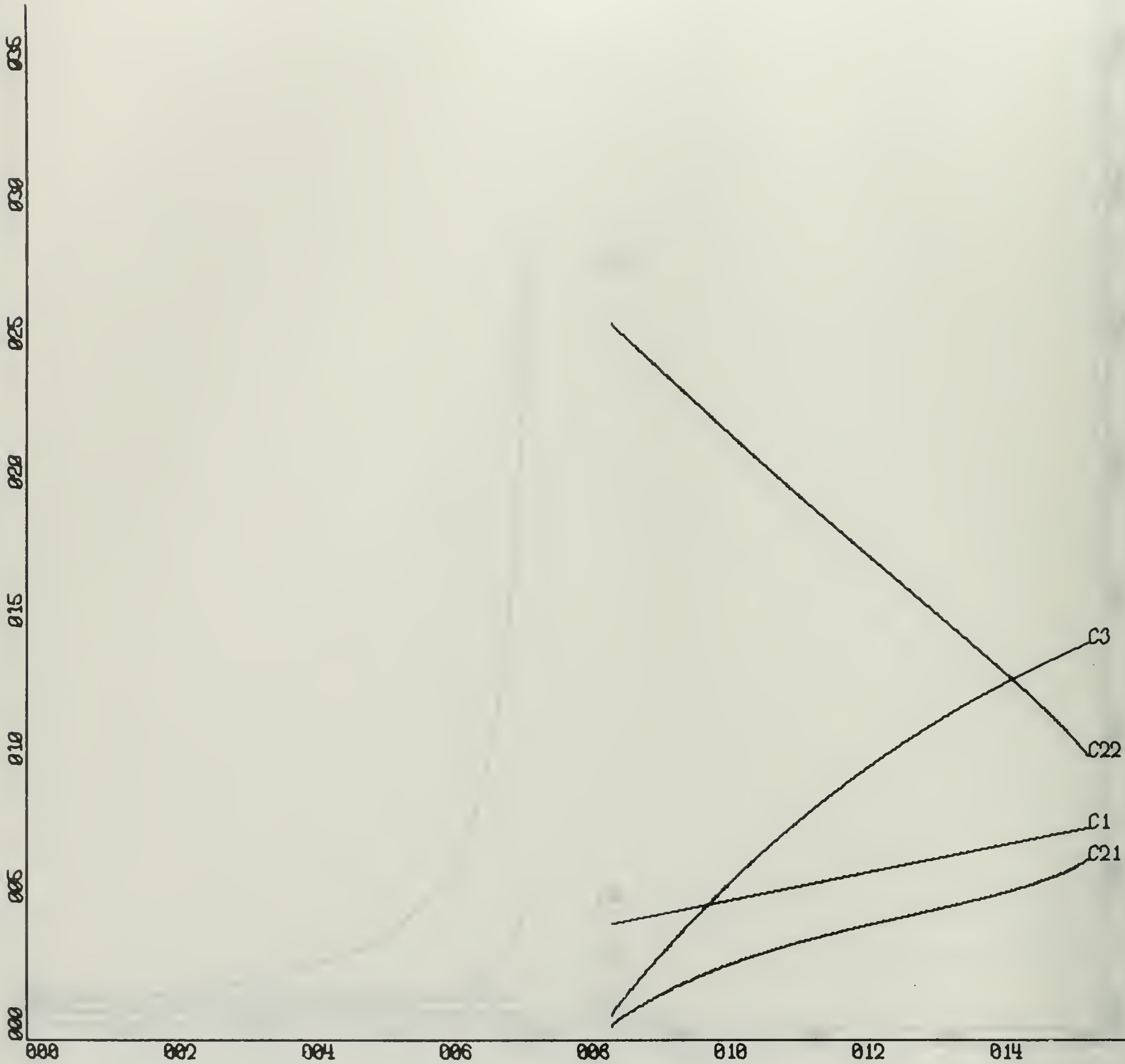
Fig. 9.a. Capacitor Values for R_1 of 1 Megohms.



-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF R2(1), R2(2), AND R3 VS TIME CONSTANT, A, FOR CONSTANT R1

Fig. 9.b. Resistor Values for R_1 of 1 Megohms.



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 5.00E-06 UNITS/INCH.

LILLIS, J. W. THESIS PLOT OF C1,C2(1), C2(2), AND C3 VS TIME CONSTANT, A, FOR CONST. R1

Fig. 10.a. Capacitor Values for R_1 of 100 Megohms.

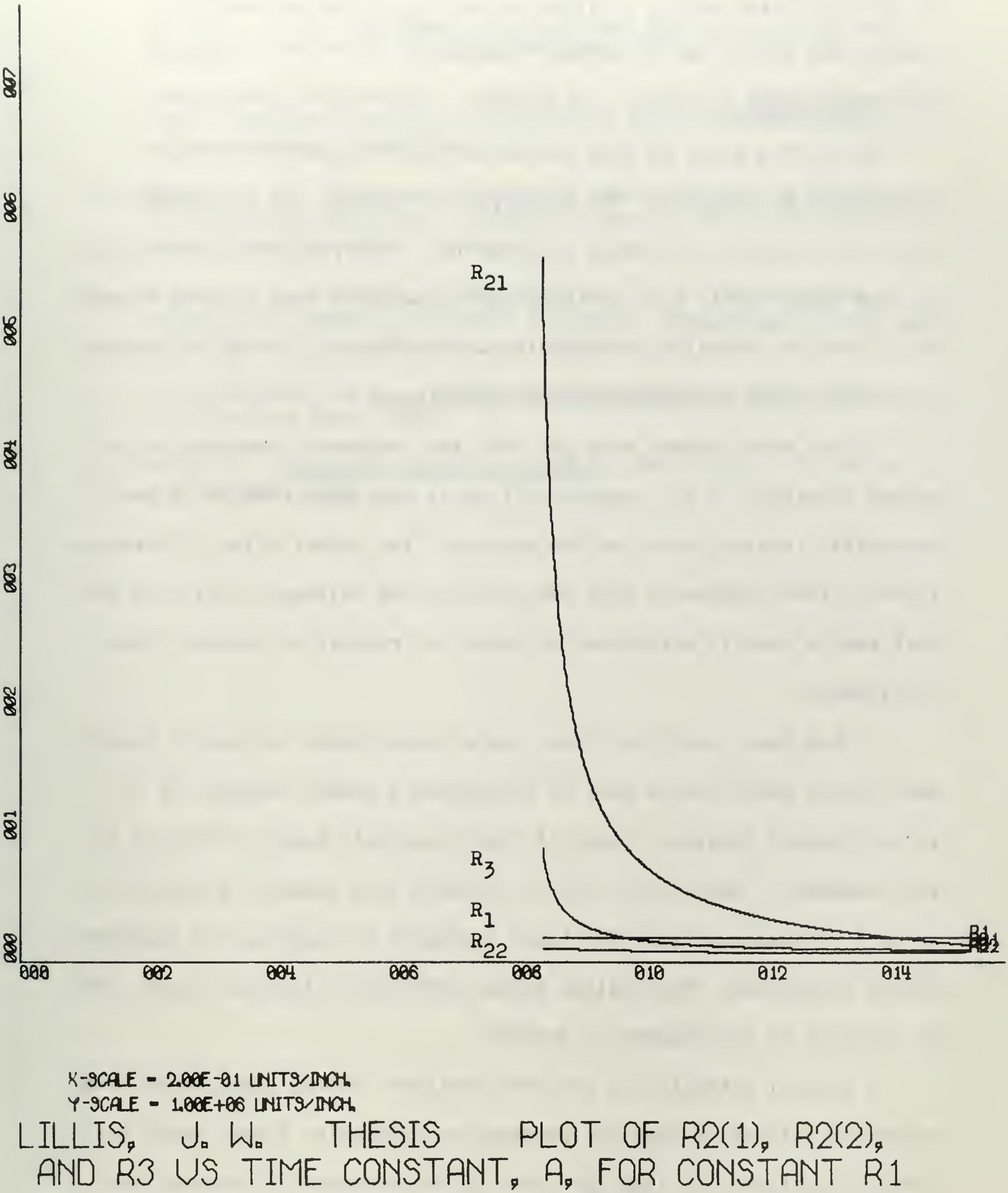


Fig. 10.b. Resistor Values for R_1 of 100 Megohms.

synthesizing that transfer function. Inputting additional values for R_1 to the program merely provides one with a convenient means of scaling the entire set of network components.

10. Conclusions.

Up to this point we have considered ladder networks made up entirely of RC elements. The principles of duality can be applied if we wish to consider RL ladder networks too. The procedure outlined can be used equally well for synthesizing RL networks from a given current-ratio transfer function by replacing all resistances of the RC network by inductors and all capacitors by conductances.

It has been assumed thus far that any impedance connected to the output terminals of the network will be of such magnitude as to have negligible loading effect on the network. The actual effect of various types of load impedances upon the currents and voltages within the network can be readily determined by using the results of section 4 and continuants.

It has been shown that there are a large number of sets of component values which can be used to synthesize a ladder network for a given transfer function. Each of these sets will have a different input impedance. Eq. (4.2) could be included in a computer program for network synthesis to give the input impedance for each set of component values determined. The scaling scheme previously discussed could also be included in the program as desired.

A logical extension of the work outlined in this paper would seem to be the writing of computer programs to synthesize higher order systems, investigate the loading effect on the networks for various network terminations, determine input and output impedances of the networks for each set of component values, and perhaps develop and program the equations for synthesizing RL networks.

BIBLIOGRAPHY

1. Bartlett, A. C. The Theory of Electrical Artificial Lines and Filters. New York: J. Wiley and Sons, 1931.

2. Control Data Corporation. FORTRAN 63 / Reference Manual, Control Data 1604/1604-A Computer, Publication Number 60052900, Revision A. Minneapolis: June 1964.

3. Parker, S. R., "Effects of Component Variation Upon the Behavior of Electrical Circuits". University Microfilms, Order Number 64-11, 238; Ann Arbor, Michigan.

4. Parker, S. R., Peskin, E., and Chirlian, P. M. "Continuants, Signal Flow Graphs, and Ladder Networks," Proceedings of the IEEE, Vol. 54, No. 3, March 1966, pp. 422-423.

5. Van Valkenburg, M. E. Modern Network Synthesis. New York: J. Wiley and Sons, 1960.

6. Wells, W. Advanced Course in Algebra. New York: D. C. Heath and Company, 1904.

APPENDIX I

THEOREMS OF CONTINUANTS

The following theorems are taken from the text by Bartlett.[1]

1. $k(a_1,a_2,..a_n) = k(a_n,a_{n-1},...a_2,a_1)$

2. $k(a_1,a_2,...a_n) = a_1k(a_2,...a_n) + k(a_3,...a_n)$

3. $k(a_1,...a_n) = k(a_1,...a_p)k(a_{p+1}...a_n)$

   $+ k(a_1,...a_{p-1})k(a_{p+2}...a_n)$

4. $k(a_1,...a_n)k(a_2,...a_{n-1}) = k(a_1,...a_{n-1})k(a_2,...a_n)$

   $+ (-1)^n$

5. $k(a_1,...a_n) = k(a_1,...a_{r-1})k(a_r,...a_n)$

   $+k(a_1,...a_{r-2})k(a_{r+1},...a_n)$

6. $k(0,a_2,...a_n) = k(a_3,...a_n)$

[1] A. C. Bartlett., The Theory of Electrical Artificial Lines and Filters (New York: J. Wiley and Sons, 1931), p. 47

APPENDIX II

PROGRAM SYNTHNET

```
      PROGRAM SYNTHNET
      DIMENSION DR(50),R2(2),C2(2)
      DO 999 J=1,50
  999 DR(J)=0.0
      DO 998 K=1,2
      R2(K)=0.0
  998 C2(K)=0.0
      READ 50,DK1,DK2,DK3.STARTA,STEPA,STOPA
   50 FORMAT(6F10)
      READ 60,DR(1),DR(2),DR(3),DR(4),DR(5),DR(6),DR(7),DR(8)
   60 FORMAT(8F10)
      A=STARTA
  100 Y=DK2/A-A-(2.*DK1)/(A**2)
      IF(Y) 101,101,102
  101 A=A+STEPA
      IF(STOPA-A) 125,125,100
  102 X=DK3-2.*A-DK1/(A**2)
      IF(X) 101,101,103
  103 T13=X-Y
      IF(T13)101,101.104
  104 W=(Y**2)-(4.*DK1*T13)/(A**2)
      IF(W) 101,105,105
  105 CONTINUE
      DO 116 I=1,8
      IF(DR(I))125,101,115
  115 R1=DR(I)
      C3=T13/R1
      C1=A/R1
      R3=A/C3
      R2(1)=(Y+SQRTF(W))/(2.*C3)
      C2(1)=DK1/((A**2)*R2(1))
      R2(2)=(Y-SQRTF(W))/(2.*C3)
      IF(R2(2))107.109,106
  106 C2(2)=DK1/((A**2)*R2(2))
      GO TO 111
  107 PRINT 108
  108 FORMAT(///,10X,17HR2(2) IS NEGATIVE)
      GO TO 113
  109 PRINT 110
  110 FORMAT (///,10X,13HR2(2) IS ZERO)
      GO TO 113
  111 PRINT 112,A,R1,R2(2).R3,C1,C2(2),C3
  112 FORMAT (///,10X.2HA=,E11.4,10X,3HR1=,E11.4,10X,6HR2(2)=,E11.4,
     110X,3HR3=,E11.4,/,33X,3HC1=.E11.4,10X,6HC2(2)=,E11.4,10X,3HC3=,
     2E11.4)
      DCK1=R1*C1*R2(2)*C2(2)*R3*C3
      DCK2=R1*C1*R2(2)*C2(2)+R1*C1*R2(2)*C3+R1*C1*R3*C3+R1*C2(2)*R3*C3
     1+R2(2)*C2(2)*R3*C3
      DCK3=R1*C1+R1*C2(2)+R1*C3+R2(2)*C2(2)+R2(2)*C3+R3*C3
      PRINT 120,DCK1.DCK2.DCK3
  120 FORMAT(/,33X.5HDCK1=.E11.4,8X.5HDCK2=,E11.4,11X,5HDCK3=,E11.4)
      PRINT 121,DK1.DK2,DK3
  121 FORMAT(33X,4HDK1=,E11.4,9X.4HDK2=,E11.4.12X,4HDK3=,E11.4)
  113 PRINT 114,A.R1,R2(1),R3,C1,C2(1).C3
  114 FORMAT (/,10X,2HA=,E11.4.10X.3HR1=,E11.4,10X,6HR2(1)=,E11.4.
     110X,3HR3=,E11.4,/.33X.3HC1=.E11.4,10X,6HC2(1)=,E11.4,10X,3HC3=,
```

```
    2F11.4)
     DCK1=R1*C1*R2(1)*C2(1)*R3*C3
     DCK2=R1*C1*R2(1)*C2(1)+R1*C1*R2(1)*C3+R1*C1*R3*C3+R1*C2(1)*R3*C3
    1+R2(1)*C2(1)*R3*C3
     DCK3=R1*C1+R1*C2(1)+R1*C3+R2(1)*C2(1)+R2(1)*C3+R3*C3
     PRINT 120,DCK1,DCK2,DCK3
     PRINT 121,DK1,DK2,DK3
 116 CONTINUE
     GO TO 101
 125 STOP
     END
```

```
PROGRAM NAMES
00000  ESIDENT          02146 ERRDUMP*          02771 XITDUMP*
75644 SYNTHNET          75607 SQRTF             75604 Q8QPAUSE
74427 Q8QGINTY          73271 Q8QGOTTY          73121 Q8QENTRY
73047 Q8QIBJOB          73035 Q8QLOADA          72550 Q8QFORMS

LABELED COMMON        NONE

NUMBERED COMMON       NONE

PROGRAM ENTRY POINTS
01263 READ*             01270 WRITE*            00737 SELECT*
01216 REMOVE*           01025 DETECT*           02125 GETCH*
02132 CHKSTD*           03220 LOADER*           03221 RELOCOM*
03172 RELOAD*           03106 LIBREW*           03025 CLBNBCD*
02762 EXIT*             02141 ERROR*            01252 MODIRET*
00463 RCDBN*            00371 OPCOM*            02373 RECRET*
02414 RECLIM*           00606 FLAGTST*          00253 MEMREC*
02104 CBF*              02105 CBF1*             02106 CBF2*
02113 CBF7*             00776 IOSLECT*          01632 MRW600*
05547 DK1*              05141 LMSPCH*           01177 ILMF*
02114 ECH*              01112 EXSEN*            00223 SETCLK*
00100 DATE*             00063 SIEOF*            03100 LIBPSIT*
76024 SYNTHNET          75610 SQRTF             75607 Q8QSQRTF
75605 Q8QSTOPS          75604 Q8QPAUSE          74427 Q8QINGIN
74513 Q8QGINTY          75266 Q8QENGIN          73271 Q8QINGOT
73346 Q8QGOTTY          74206 Q8QENGOT          73121 Q8QENTRY
73124 Q8QEXITS          73147 Q8QLPSEN          73150 Q8QGNSET
73153 Q8QFTSEN          73154 Q8QFTSET          73157 Q8QIOSEN
73160 Q8QIOSET          73163 Q8QGNSEN          73202 Q8QLPSET
73247 Q8QGNTAB          73260 Q8QGTTAB          73263 Q8QBUTAB
73047 Q8QIBJOB          73035 Q8QLOADA          72550 Q8QIFORM
72570 Q8QFORMS          72763 Q8QGLIST
```

```
A= 8.5000E-01     R1= 2.0000E 05         R2(2)= 1.1989E 05      R3= 9.3492E 05
                  C1= 4.2500E-06         C2(2)= 2.5975E-05      C3= 9.0917E-07

                  DCK1= 2.2500E 00       DCK2= 1.0525E 01       DCK3= 1.0300E 01
                  DK1= 2.2500E 00        DK2= 1.0525E 01        DK3= 1.0300E 01

A= 8.5000E-01     R1= 2.0000E 05         R2(1)= 5.7140E 06      R3= 9.3492E 05
                  C1= 4.2500E-06         C2(1)= 5.4501E-07      C3= 9.0917E-07

                  DCK1= 2.2500E 00       DCK2= 1.0525E 01       DCK3= 1.0300E 01
                  DK1= 2.2500E 00        DK2= 1.0525E 01        DK3= 1.0300E 01


A= 8.5000E-01     R1= 2.5000E 05         R2(2)= 1.4987E 05      R3= 1.1686E 06
                  C1= 3.4000E-06         C2(2)= 2.0780E-05      C3= 7.2734E-07

                  DCK1= 2.2500E 00       DCK2= 1.0525E 01       DCK3= 1.0300E 01
                  DK1= 2.2500E 00        DK2= 1.0525E 01        DK3= 1.0300E 01

A= 8.5000E-01     R1= 2.5000E 05         R2(1)= 7.1425E 06      R3= 1.1686E 06
                  C1= 3.4000E-06         C2(1)= 4.3601E-07      C3= 7.2734E-07

                  DCK1= 2.2500E 00       DCK2= 1.0525E 01       DCK3= 1.0300E 01
                  DK1= 2.2500E 00        DK2= 1.0525E 01        DK3= 1.0300E 01
```

```
A= 8.5000E-01        R1= 1.0000E 06        R2(2)= 5.9946E 05     R3= 4.6746E 06
                     C1= 8.5000E-07        C2(2)= 5.1950E-06     C3= 1.8183E-07

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01

A= 8.5000E-01        R1= 1.0000E 06        R2(1)= 2.8570E 07     R3= 4.6746E 06
                     C1= 8.5000E-07        C2(1)= 1.0900E-07     C3= 1.8183E-07

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01


A= 8.5000E-01        R1= 1.5000E 05        R2(2)= 8.9919E 04     R3= 7.0119E 05
                     C1= 5.6667E-06        C2(2)= 3.4633E-05     C3= 1.2122E-06

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01

A= 8.5000E-01        R1= 1.5000E 05        R2(1)= 4.2855E 06     R3= 7.0119E 05
                     C1= 5.6667E-06        C2(1)= 7.2668E-07     C3= 1.2122E-06

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01


A= 8.5000E-01        R1= 5.0000E-01        R2(2)= 2.9973E-01     R3= 2.3373E 00
                     C1= 1.7000E 00        C2(2)= 1.0390E 01     C3= 3.6367E-01

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01

A= 8.5000E-01        R1= 5.0000E-01        R2(1)= 1.4285E 01     R3= 2.3373E 00
                     C1= 1.7000E 00        C2(1)= 2.1800E-01     C3= 3.6367E-01

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01


A= 8.5000E-01        R1= 1.0000E 08        R2(2)= 5.9946E 07     R3= 4.6746E 08
                     C1= 8.5000E-09        C2(2)= 5.1950E-08     C3= 1.8183E-09

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01

A= 8.5000E-01        R1= 1.0000E 08        R2(1)= 2.8570E 09     R3= 4.6746E 08
                     C1= 8.5000E-09        C2(1)= 1.0900E-09     C3= 1.8183E-09

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01


A= 9.0000E-01        R1= 2.0000E 05        R2(2)= 1.1161E 05     R3= 3.7241E 05
                     C1= 4.5000E-06        C2(2)= 2.4843E-05     C3= 2.4167E-06

                     DCK1= 2.2500E 00      DCK2= 1.0525E 01      DCK3= 1.0300E 01
                     DK1= 2.2500E 00       DK2= 1.0525E 01       DK3= 1.0300E 01

A= 9.0000E-01        R1= 2.0000E 05        R2(1)= 2.0560E 06     R3= 3.7241E 05
                     C1= 4.5000E-06        C2(1)= 1.3511E-06     C3= 2.4167E-06
```

	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 2.5000E 05 C1= 3.6000E-06	R2(2)= 1.3976E 05 C2(2)= 1.9875E-05	R3= 4.6552E 05 C3= 1.9333E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 2.5000E 05 C1= 3.6000E-06	R2(1)= 2.5700E 06 C2(1)= 1.0808E-06	R3= 4.6552E 05 C3= 1.9333E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 1.0000E 06 C1= 9.0000E-07	R2(2)= 5.5906E 05 C2(2)= 4.9687E-06	R3= 1.8621E 06 C3= 4.8333E-07
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 1.0000E 06 C1= 9.0000E-07	R2(1)= 1.0280E 07 C2(1)= 2.7021E-07	R3= 1.8621E 06 C3= 4.8333E-07
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 1.5000E 05 C1= 6.0000E-06	R2(2)= 8.3859E 04 C2(2)= 3.3125E-05	R3= 2.7931E 05 C3= 3.2222E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 1.5000E 05 C1= 6.0000E-06	R2(1)= 1.5420E 06 C2(1)= 1.8014E-06	R3= 2.7931E 05 C3= 3.2222E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 5.0000E-01 C1= 1.8000E 00	R2(2)= 2.7953E-01 C2(2)= 9.9374E 00	R3= 9.3103E-01 C3= 9.6667E-01
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 5.0000E-01 C1= 1.8000E 00	R2(1)= 5.1400E 00 C2(1)= 5.4042E-01	R3= 9.3103E-01 C3= 9.6667E-01
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 9.0000E-01	R1= 1.0000E 08 C1= 9.0000E-09	R2(2)= 5.5906E 07 C2(2)= 4.9687E-08	R3= 1.8621E 08 C3= 4.8333E-09
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01

	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.0000E-01	R1= 1.0000E 08	R2(1)= 1.0280E 09	R3= 1.8621E 08
	C1= 9.0000E-09	C2(1)= 2.7021E-09	C3= 4.8333E-09
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 2.0000E 05	R2(2)= 1.0517E 05	R3= 2.4865E 05
	C1= 4.7500E-06	C2(2)= 2.3705E-05	C3= 3.8206E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 2.0000E 05	R2(1)= 1.2409E 06	R3= 2.4865E 05
	C1= 4.7500E-06	C2(1)= 2.0091E-06	C3= 3.8206E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 2.5000E 05	R2(2)= 1.3146E 05	R3= 3.1081E 05
	C1= 3.8000E-06	C2(2)= 1.8964E-05	C3= 3.0565E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 2.5000E 05	R2(1)= 1.5511E 06	R3= 3.1081E 05
	C1= 3.8000E-06	C2(1)= 1.6073E-06	C3= 3.0565E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 1.0000E 06	R2(2)= 5.2586E 05	R3= 1.2432E 06
	C1= 9.5000E-07	C2(2)= 4.7410E-06	C3= 7.6413E-07
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 1.0000E 06	R2(1)= 6.2044E 06	R3= 1.2432E 06
	C1= 9.5000E-07	C2(1)= 4.0182E-07	C3= 7.6413E-07
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 1.5000E 05	R2(2)= 7.8879E 04	R3= 1.8649E 05
	C1= 6.3333E-06	C2(2)= 3.1607E-05	C3= 5.0942E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 9.5000E-01	R1= 1.5000E 05	R2(1)= 9.3066E 05	R3= 1.8649E 05
	C1= 6.3333E-06	C2(1)= 2.6788E-06	C3= 5.0942E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01

A= 9.5000E-01	R1= 5.0000E-01	R^2(2)= 2.6293E-01	R^3= 6.2162E-01
	C1= 1.9000E 00	C^2(2)= 9.4820E 00	C^3= 1.5283E 00
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 9.5000E-01	R1= 5.0000E-01	R^2(1)= 3.1022E 00	R^3= 6.2162E-01
	C1= 1.9000E 00	C^2(1)= 8.0364E-01	C^3= 1.5283E 00
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 9.5000E-01	R1= 1.0000E 08	R^2(2)= 5.2586E 07	R^3= 1.2432E 08
	C1= 9.5000E-09	C^2(2)= 4.7410E-08	C^3= 7.6413E-09
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 9.5000E-01	R1= 1.0000E 08	R^2(1)= 6.2044E 08	R^3= 1.2432E 08
	C1= 9.5000E-09	C^2(1)= 4.0182E-09	C^3= 7.6413E-09
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 1.0000E 00	R1= 2.0000E 05	R^2(2)= 9.9688E 04	R^3= 1.9512E 05
	C1= 5.0000E-06	C^2(2)= 2.2571E-05	C^3= 5.1250E-06
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 1.0000E 00	R1= 2.0000E 05	R^2(1)= 8.8080E 05	R^3= 1.9512E 05
	C1= 5.0000E-06	C^2(1)= 2.5545E-06	C^3= 5.1250E-06
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 1.0000E 00	R1= 2.5000E 05	R^2(2)= 1.2461E 05	R^3= 2.4390E 05
	C1= 4.0000E-06	C^2(2)= 1.8056E-05	C^3= 4.1000E-06
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 1.0000E 00	R1= 2.5000E 05	R^2(1)= 1.1010E 06	R^3= 2.4390E 05
	C1= 4.0000E-06	C^2(1)= 2.0436E-06	C^3= 4.1000E-06
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 1.0000E 00	R1= 1.0000E 06	R^2(2)= 4.9844E 05	R^3= 9.7561E 05
	C1= 1.0000E-06	C^2(2)= 4.5141E-06	C^3= 1.0250E-06
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01
	DK1= 2.2500E 00	DK^2= 1.0525E 01	DK^3= 1.0300E 01
A= 1.0000E 00	R1= 1.0000E 06	R^2(1)= 4.4040E 06	R^3= 9.7561E 05
	C1= 1.0000E-06	C^2(1)= 5.1090E-07	C^3= 1.0250E-06
	DCK1= 2.2500E 00	DCK^2= 1.0525E 01	DCK^3= 1.0300E 01

	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.0000E 00	R1= 1.5000E 05 C1= 6.6667E-06	R2(2)= 7.4766E 04 C2(2)= 3.0094E-05	R3= 1.4634E 05 C3= 6.8333E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0000E 00	R1= 1.5000E 05 C1= 6.6667E-06	R2(1)= 6.6060E 05 C2(1)= 3.4060E-06	R3= 1.4634E 05 C3= 6.8333E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0000E 00	R1= 5.0000E-01 C1= 2.0000E 00	R2(2)= 2.4922E-01 C2(2)= 9.0282E 00	R3= 4.8780E-01 C3= 2.0500E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0000E 00	R1= 5.0000E-01 C1= 2.0000E 00	R2(1)= 2.2020E 00 C2(1)= 1.0218E 00	R3= 4.8780E-01 C3= 2.0500E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0000E 00	R1= 1.0000E 08 C1= 1.0000E-08	R2(2)= 4.9844E 07 C2(2)= 4.5141E-08	R3= 9.7561E 07 C3= 1.0250E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0000E 00	R1= 1.0000E 08 C1= 1.0000E-08	R2(1)= 4.4040E 08 C2(1)= 5.1090E-09	R3= 9.7561E 07 C3= 1.0250E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 2.0000E 05 C1= 5.2500E-06	R2(2)= 9.5157E 04 C2(2)= 2.1447E-05	R3= 1.6574E 05 C3= 6.3350E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 2.0000E 05 C1= 5.2500E-06	R2(1)= 6.7708E 05 C2(1)= 3.0141E-06	R3= 1.6574E 05 C3= 6.3350E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 2.5000E 05 C1= 4.2000E-06	R2(2)= 1.1895E 05 C2(2)= 1.7157E-05	R3= 2.0718E 05 C3= 5.0680E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01

A= 1.0500E 00	R1= 2.5000E 05 C1= 4.2000E-06	R2(1)= 8.4636E 05 C2(1)= 2.4113E-06	R3= 2.0718E 05 C3= 5.0680E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 1.0000E 06 C1= 1.0500E-06	R2(2)= 4.7579E 05 C2(2)= 4.2894E-06	R3= 8.2872E 05 C3= 1.2670E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 1.0000E 06 C1= 1.0500E-06	R2(1)= 3.3854E 06 C2(1)= 6.0282E-07	R3= 8.2872E 05 C3= 1.2670E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 1.5000E 05 C1= 7.0000E-06	R2(2)= 7.1368E 04 C2(2)= 2.8596E-05	R3= 1.2431E 05 C3= 8.4467E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300F 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 1.5000E 05 C1= 7.0000E-06	R2(1)= 5.0781E 05 C2(1)= 4.0188E-06	R3= 1.2431E 05 C3= 8.4467E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 5.0000E-01 C1= 2.1000E 00	R2(2)= 2.3789E-01 C2(2)= 8.5787E 00	R3= 4.1436E-01 C3= 2.5340E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 5.0000E-01 C1= 2.1000E 00	R2(1)= 1.6927E 00 C2(1)= 1.2056E 00	R3= 4.1436E-01 C3= 2.5340E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 1.0000E 08 C1= 1.0500E-08	R2(2)= 4.7579E 07 C2(2)= 4.2894E-08	R3= 8.2872E 07 C3= 1.2670E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.0500E 00	R1= 1.0000E 08 C1= 1.0500E-08	R2(1)= 3.3854E 08 C2(1)= 6.0282E-09	R3= 8.2872E 07 C3= 1.2670E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 2.0000E 05 C1= 5.5000E-06	R2(2)= 9.1435E 04 C2(2)= 2.0337E-05	R3= 1.4752E 05 C3= 7.4566E-06

	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 2.0000E 05 C1= 5.5000E-06	R2(1)= 5.4547E 05 C2(1)= 3.4090E-06	R3= 1.4752E 05 C3= 7.4566E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 2.5000E 05 C1= 4.4000E-06	R2(2)= 1.1429E 05 C2(2)= 1.6270E-05	R3= 1.8440E 05 C3= 5.9653E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 2.5000E 05 C1= 4.4000E-06	R2(1)= 6.8184E 05 C2(1)= 2.7272E-06	R3= 1.8440E 05 C3= 5.9653E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 1.0000E 06 C1= 1.1000E-06	R2(2)= 4.5718E 05 C2(2)= 4.0674E-06	R3= 7.3760E 05 C3= 1.4913E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 1.0000E 06 C1= 1.1000E-06	R2(1)= 2.7274E 06 C2(1)= 6.8180E-07	R3= 7.3760E 05 C3= 1.4913E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 1.5000E 05 C1= 7.3333E-06	R2(2)= 6.8576E 04 C2(2)= 2.7116E-05	R3= 1.1064E 05 C3= 9.9421E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 1.5000E 05 C1= 7.3333E-06	R2(1)= 4.0910E 05 C2(1)= 4.5453E-06	R3= 1.1064E 05 C3= 9.9421E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 5.0000E-01 C1= 2.2000E 00	R2(2)= 2.2859E-01 C2(2)= 8.1348E 00	R3= 3.6880E-01 C3= 2.9826E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.1000E 00	R1= 5.0000E-01 C1= 2.2000E 00	R2(1)= 1.3637E 00 C2(1)= 1.3636E 00	R3= 3.6880E-01 C3= 2.9826E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01

```
A= 1.1000E 00      R1= 1.0000E 08       R2(2)= 4.5718E 07     R3= 7.3760E 07
                   C1= 1.1000E-08       C2(2)= 4.0674E-08     C3= 1.4913E-08

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01
A= 1.1000E 00      R1= 1.0000E 08       R2(1)= 2.7274E 08     R3= 7.3760E 07
                   C1= 1.1000E-08       C2(1)= 6.8180E-09     C3= 1.4913E-08

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01


A= 1.1500E 00      R1= 2.0000E 05       R2(2)= 8.8418E 04     R3= 1.3536E 05
                   C1= 5.7500E-06       C2(2)= 1.9242E-05     C3= 8.4957E-06

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01
A= 1.1500E 00      R1= 2.0000E 05       R2(1)= 4.5297E 05     R3= 1.3536E 05
                   C1= 5.7500E-06       C2(1)= 3.7559E-06     C3= 8.4957E-06

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01


A= 1.1500E 00      R1= 2.5000E 05       R2(2)= 1.1052E 05     R3= 1.6920E 05
                   C1= 4.6000E-06       C2(2)= 1.5393E-05     C3= 6.7966E-06

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01
A= 1.1500E 00      R1= 2.5000E 05       R2(1)= 5.6622E 05     R3= 1.6920E 05
                   C1= 4.6000E-06       C2(1)= 3.0047E-06     C3= 6.7966E-06

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01


A= 1.1500E 00      R1= 1.0000E 06       R2(2)= 4.4209E 05     R3= 6.7681E 05
                   C1= 1.1500E-06       C2(2)= 3.8483E-06     C3= 1.6991E-06

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01
A= 1.1500E 00      R1= 1.0000E 06       R2(1)= 2.2649E 06     R3= 6.7681E 05
                   C1= 1.1500E-06       C2(1)= 7.5118E-07     C3= 1.6991E-06

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01


A= 1.1500E 00      R1= 1.5000E 05       R2(2)= 6.6314E 04     R3= 1.0152E 05
                   C1= 7.6667E-06       C2(2)= 2.5656E-05     C3= 1.1328E-05

                   DCK1= 2.2500E 00     DCK2= 1.0525E 01      DCK3= 1.0300E 01
                   DK1= 2.2500E 00      DK2= 1.0525E 01       DK3= 1.0300E 01
A= 1.1500E 00      R1= 1.5000E 05       R2(1)= 3.3973E 05     R3= 1.0152E 05
                   C1= 7.6667E-06       C2(1)= 5.0079E-06     C3= 1.1328E-05
```

	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.1500E 00	R1= 5.0000E-01	R2(2)= 2.2105E-01	R3= 3.3840E-01
	C1= 2.3000E 00	C2(2)= 7.6967E 00	C3= 3.3983E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.1500E 00	R1= 5.0000E-01	R2(1)= 1.1324E 00	R3= 3.3840E-01
	C1= 2.3000E 00	C2(1)= 1.5024E 00	C3= 3.3983E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.1500E 00	R1= 1.0000E 08	R2(2)= 4.4209E 07	R3= 6.7681E 07
	C1= 1.1500E-08	C2(2)= 3.8483E-08	C3= 1.6991E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.1500E 00	R1= 1.0000E 08	R2(1)= 2.2649E 08	R3= 6.7681E 07
	C1= 1.1500E-08	C2(1)= 7.5118E-09	C3= 1.6991E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 2.0000E 05	R2(2)= 8.6040E 04	R3= 1.2687E 05
	C1= 6.0000E-06	C2(2)= 1.8160E-05	C3= 9.4583E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 2.0000E 05	R2(1)= 3.8400E 05	R3= 1.2687E 05
	C1= 6.0000E-06	C2(1)= 4.0690E-06	C3= 9.4583E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 2.5000E 05	R2(2)= 1.0755E 05	R3= 1.5859E 05
	C1= 4.8000E-06	C2(2)= 1.4528E-05	C3= 7.5667E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 2.5000E 05	R2(1)= 4.8001E 05	R3= 1.5859E 05
	C1= 4.8000E-06	C2(1)= 3.2552E-06	C3= 7.5667E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 1.0000E 06	R2(2)= 4.3020E 05	R3= 6.3436E 05
	C1= 1.2000E-06	C2(2)= 3.6320E-06	C3= 1.8917E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01

	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 1.0000E 06	R2(1)= 1.9200E 06	R3= 6.3436E 05
	C1= 1.2000E-06	C2(1)= 8.1379E-07	C3= 1.8917E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 1.5000E 05	R2(2)= 6.4530E 04	R3= 9.5154E 04
	C1= 8.0000E-06	C2(2)= 2.4214E-05	C3= 1.2611E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 1.5000E 05	R2(1)= 2.8800E 05	R3= 9.5154E 04
	C1= 8.0000E-06	C2(1)= 5.4253E-06	C3= 1.2611E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 5.0000E-01	R2(2)= 2.1510E-01	R3= 3.1718E-01
	C1= 2.4000E 00	C2(2)= 7.2641E 00	C3= 3.7833E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 5.0000E-01	R2(1)= 9.6001E-01	R3= 3.1718E-01
	C1= 2.4000E 00	C2(1)= 1.6276E 00	C3= 3.7833E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 1.0000E 08	R2(2)= 4.3020E 07	R3= 6.3436E 07
	C1= 1.2000E-08	C2(2)= 3.6320E-08	C3= 1.8917E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2000E 00	R1= 1.0000E 08	R2(1)= 1.9200E 08	R3= 6.3436E 07
	C1= 1.2000E-08	C2(1)= 8.1379E-09	C3= 1.8917E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 2.0000E 05	R2(2)= 8.4263E 04	R3= 1.2077E 05
	C1= 6.2500E-06	C2(2)= 1.7089E-05	C3= 1.0350E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 2.0000E 05	R2(1)= 3.3023E 05	R3= 1.2077E 05
	C1= 6.2500E-06	C2(1)= 4.3606E-06	C3= 1.0350E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01

A= 1.2500E 00	R1= 2.5000E 05	R2(2)= 1.0533E 05	R3= 1.5097E 05
	C1= 5.0000E-06	C2(2)= 1.3672E-05	C3= 8.2800E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 2.5000E 05	R2(1)= 4.1279E 05	R3= 1.5097E 05
	C1= 5.0000E-06	C2(1)= 3.4885E-06	C3= 8.2800E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 1.0000E 06	R2(2)= 4.2131E 05	R3= 6.0386E 05
	C1= 1.2500E-06	C2(2)= 3.4179E-06	C3= 2.0700E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 1.0000E 06	R2(1)= 1.6512E 06	R3= 6.0386E 05
	C1= 1.2500E-06	C2(1)= 8.7212E-07	C3= 2.0700E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 1.5000E 05	R2(2)= 6.3197E 04	R3= 9.0580E 04
	C1= 8.3333E-06	C2(2)= 2.2786E-05	C3= 1.3800E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 1.5000E 05	R2(1)= 2.4767E 05	R3= 9.0580E 04
	C1= 8.3333E-06	C2(1)= 5.8141E-06	C3= 1.3800E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 5.0000E-01	R2(2)= 2.1066E-01	R3= 3.0193E-01
	C1= 2.5000E 00	C2(2)= 6.8358E 00	C3= 4.1400E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 5.0000E-01	R2(1)= 8.2558E-01	R3= 3.0193E-01
	C1= 2.5000E 00	C2(1)= 1.7442E 00	C3= 4.1400E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 1.0000E 08	R2(2)= 4.2131E 07	R3= 6.0386E 07
	C1= 1.2500E-08	C2(2)= 3.4179E-08	C3= 2.0700E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.2500E 00	R1= 1.0000E 08	R2(1)= 1.6512E 08	R3= 6.0386E 07
	C1= 1.2500E-08	C2(1)= 8.7212E-09	C3= 2.0700E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01

```
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01

A= 1.3000E 00    R1= 2.0000E 05          R2(2)= 8.3083E 04         R3= 1.1632E 05
                 C1= 6.5000E-06          C2(2)= 1.6024E-05         C3= 1.1176E-05

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01

A= 1.3000E 00    R1= 2.0000E 05          R2(1)= 2.8676E 05         R3= 1.1632E 05
                 C1= 6.5000E-06          C2(1)= 4.6427E-06         C3= 1.1176E-05

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01


A= 1.3000E 00    R1= 2.5000E 05          R2(2)= 1.0385E 05         R3= 1.4540E 05
                 C1= 5.2000E-06          C2(2)= 1.2820E-05         C3= 8.9408E-06

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01

A= 1.3000E 00    R1= 2.5000E 05          R2(1)= 3.5846E 05         R3= 1.4540E 05
                 C1= 5.2000E-06          C2(1)= 3.7142E-06         C3= 8.9408E-06

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01


A= 1.3000E 00    R1= 1.0000E 06          R2(2)= 4.1542E 05         R3= 5.8160E 05
                 C1= 1.3000E-06          C2(2)= 3.2049E-06         C3= 2.2352E-06

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01

A= 1.3000E 00    R1= 1.0000E 06          R2(1)= 1.4338E 06         R3= 5.8160E 05
                 C1= 1.3000E-06          C2(1)= 9.2854E-07         C3= 2.2352E-06

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01


A= 1.3000E 00    R1= 1.5000E 05          R2(2)= 6.2312E 04         R3= 8.7240E 04
                 C1= 8.6667E-06          C2(2)= 2.1366E-05         C3= 1.4901E-05

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01

A= 1.3000E 00    R1= 1.5000E 05          R2(1)= 2.1507E 05         R3= 8.7240E 04
                 C1= 8.6667E-06          C2(1)= 6.1903E-06         C3= 1.4901E-05

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01


A= 1.3000E 00    R1= 5.0000E-01          R2(2)= 2.0771E-01         R3= 2.9080E-01
                 C1= 2.6000E 00          C2(2)= 6.4098E 00         C3= 4.4704E 00

                 DCK1= 2.2500E 00        DCK2= 1.0525E 01          DCK3= 1.0300E 01
                 DK1= 2.2500E 00         DK2= 1.0525E 01           DK3= 1.0300E 01
```

A= 1.3000E 00	R1= 5.0000E-01 C1= 2.6000E 00	R2(1)= 7.1691E-01 C2(1)= 1.8571E 00	R3= 2.9080E-01 C3= 4.4704E 00
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3000E 00	R1= 1.0000E 08 C1= 1.3000E-08	R2(2)= 4.1542E 07 C2(2)= 3.2049E-08	R3= 5.8160E 07 C3= 2.2352E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3000E 00	R1= 1.0000E 08 C1= 1.3000E-08	R2(1)= 1.4338E 08 C2(1)= 9.2854E-09	R3= 5.8160E 07 C3= 2.2352E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 2.0000E 05 C1= 6.7500E-06	R2(2)= 8.2537E 04 C2(2)= 1.4958E-05	R3= 1.1305E 05 C3= 1.1941E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 2.0000E 05 C1= 6.7500E-06	R2(1)= 2.5052E 05 C2(1)= 4.9280E-06	R3= 1.1305E 05 C3= 1.1941E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 2.5000E 05 C1= 5.4000E-06	R2(2)= 1.0317E 05 C2(2)= 1.1966E-05	R3= 1.4132E 05 C3= 9.5531E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 2.5000E 05 C1= 5.4000E-06	R2(1)= 3.1315E 05 C2(1)= 3.9424E-06	R3= 1.4132E 05 C3= 9.5531E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 1.0000E 06 C1= 1.3500E-06	R2(2)= 4.1268E 05 C2(2)= 2.9916E-06	R3= 5.6526E 05 C3= 2.3883E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 1.0000E 06 C1= 1.3500E-06	R2(1)= 1.2526E 06 C2(1)= 9.8560E-07	R3= 5.6526E 05 C3= 2.3883E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.3500E 00	R1= 1.5000E 05 C1= 9.0000E-06	R2(2)= 6.1903E 04 C2(2)= 1.0044E-05	R3= 8.4789E 04 C3= 1.5022E-05

	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.3500E 00	R1= 1.5000E 05	R2(1)= 1.8789E 05	R3= 8.4789E 04
	C1= 9.0000E-06	C2(1)= 6.5707E-06	C3= 1.5922E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.3500E 00	R1= 5.0000E-01	R2(2)= 2.0634E-01	R3= 2.8263E-01
	C1= 2.7000E 00	C2(2)= 5.9831E 00	C3= 4.7765E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.3500E 00	R1= 5.0000E-01	R2(1)= 6.2630E-01	R3= 2.8263E-01
	C1= 2.7000E 00	C2(1)= 1.9712E 00	C3= 4.7765E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.3500E 00	R1= 1.0000E 08	R2(2)= 4.1268E 07	R3= 5.6526E 07
	C1= 1.3500E-08	C2(2)= 2.9916E-08	C3= 2.3883E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.3500E 00	R1= 1.0000E 08	R2(1)= 1.2526E 08	R3= 5.6526E 07
	C1= 1.3500E-08	C2(1)= 9.8560E-09	C3= 2.3883E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 2.0000E 05	R2(2)= 8.2722E 04	R3= 1.1067E 05
	C1= 7.0000E-06	C2(2)= 1.3877E-05	C3= 1.2651E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 2.0000E 05	R2(1)= 2.1940E 05	R3= 1.1067E 05
	C1= 7.0000E-06	C2(1)= 5.2324E-06	C3= 1.2651E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 2.5000E 05	R2(2)= 1.0340E 05	R3= 1.3833E 05
	C1= 5.6000E-06	C2(2)= 1.1102E-05	C3= 1.0120E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 2.5000E 05	R2(1)= 2.7424E 05	R3= 1.3833E 05
	C1= 5.6000E-06	C2(1)= 4.1859E-06	C3= 1.0120E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01

A= 1.4000E 00	R1= 1.0000E 06	R2(2)= 4.1361E 05	R3= 5.5334E 05
	C1= 1.4000E-06	C2(2)= 2.7755E-06	C3= 2.5301E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 1.0000E 06	R2(1)= 1.0970E 06	R3= 5.5334E 05
	C1= 1.4000E-06	C2(1)= 1.0465E-06	C3= 2.5301E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 1.5000E 05	R2(2)= 6.2041E 04	R3= 8.3001E 04
	C1= 9.3333E-06	C2(2)= 1.8503E-05	C3= 1.6867E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 1.5000E 05	R2(1)= 1.6455E 05	R3= 8.3001E 04
	C1= 9.3333E-06	C2(1)= 6.9765E-06	C3= 1.6867E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 5.0000E-01	R2(2)= 2.0680E-01	R3= 2.7667E-01
	C1= 2.8000E 00	C2(2)= 5.5509E 00	C3= 5.0602E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 5.0000E-01	R2(1)= 5.4849E-01	R3= 2.7667E-01
	C1= 2.8000E 00	C2(1)= 2.0930E 00	C3= 5.0602E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 1.0000E 08	R2(2)= 4.1361E 07	R3= 5.5334E 07
	C1= 1.4000E-08	C2(2)= 2.7755E-08	C3= 2.5301E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4000E 00	R1= 1.0000E 08	R2(1)= 1.0970E 08	R3= 5.5334E 07
	C1= 1.4000E-08	C2(1)= 1.0465E-08	C3= 2.5301E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 2.0000E 05	R2(2)= 8.3855E 04	R3= 1.0896E 05
	C1= 7.2500E-06	C2(2)= 1.2762E-05	C3= 1.3308E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 2.0000E 05	R2(1)= 1.9180E 05	R3= 1.0896E 05
	C1= 7.2500E-06	C2(1)= 5.5796E-06	C3= 1.3308E-05

	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 2.5000E 05	R2(2)= 1.0482E 05	R3= 1.3620E 05
	C1= 5.8000E-06	C2(2)= 1.0210E-05	C3= 1.0646E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 2.5000E 05	R2(1)= 2.3975E 05	R3= 1.3620E 05
	C1= 5.8000E-06	C2(1)= 4.4636E-06	C3= 1.0646E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 1.0000E 06	R2(2)= 4.1927E 05	R3= 5.4480E 05
	C1= 1.4500E-06	C2(2)= 2.5524E-06	C3= 2.6615E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 1.0000E 06	R2(1)= 9.5900E 05	R3= 5.4480E 05
	C1= 1.4500E-06	C2(1)= 1.1159E-06	C3= 2.6615E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 1.5000E 05	R2(2)= 6.2891E 04	R3= 8.1720E 04
	C1= 9.6667E-06	C2(2)= 1.7016E-05	C3= 1.7744E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 1.5000E 05	R2(1)= 1.4385E 05	R3= 8.1720E 04
	C1= 9.6667E-06	C2(1)= 7.4394E-06	C3= 1.7744E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 5.0000E-01	R2(2)= 2.0964E-01	R3= 2.7240E-01
	C1= 2.9000E 00	C2(2)= 5.1048E 00	C3= 5.3231E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 5.0000E-01	R2(1)= 4.7950E-01	R3= 2.7240E-01
	C1= 2.9000E 00	C2(1)= 2.2318E 00	C3= 5.3231E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 1.0000E 08	R2(2)= 4.1927E 07	R3= 5.4480E 07
	C1= 1.4500E-08	C2(2)= 2.5524E-08	C3= 2.6615E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01

	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.4500E 00	R1= 1.0000E 08 C1= 1.4500E-08	R2(1)= 9.5900E 07 C2(1)= 1.1159E-08	R3= 5.4480E 07 C3= 2.6615E-08
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 2.0000E 05 C1= 7.5000E-06	R2(2)= 8.6443E 04 C2(2)= 1.1568E-05	R3= 1.0778E 05 C3= 1.3917E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 2.0000E 05 C1= 7.5000E-06	R2(1)= 1.6625E 05 C2(1)= 6.0150E-06	R3= 1.0778E 05 C3= 1.3917E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 2.5000E 05 C1= 6.0000E-06	R2(2)= 1.0805E 05 C2(2)= 9.2547E-06	R3= 1.3473E 05 C3= 1.1133E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 2.5000E 05 C1= 6.0000E-06	R2(1)= 2.0781E 05 C2(1)= 4.8120E-06	R3= 1.3473E 05 C3= 1.1133E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 1.0000E 06 C1= 1.5000E-06	R2(2)= 4.3221E 05 C2(2)= 2.3137E-06	R3= 5.3892E 05 C3= 2.7833E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 1.0000E 06 C1= 1.5000E-06	R2(1)= 8.3126E 05 C2(1)= 1.2030E-06	R3= 5.3892E 05 C3= 2.7833E-06
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 1.5000E 05 C1= 1.0000E-05	R2(2)= 6.4832E 04 C2(2)= 1.5424E-05	R3= 8.0838E 04 C3= 1.8556E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01
A= 1.5000E 00	R1= 1.5000E 05 C1= 1.0000E-05	R2(1)= 1.2469E 05 C2(1)= 8.0200E-06	R3= 8.0838E 04 C3= 1.8556E-05
	DCK1= 2.2500E 00 DK1= 2.2500E 00	DCK2= 1.0525E 01 DK2= 1.0525E 01	DCK3= 1.0300E 01 DK3= 1.0300E 01

A= 1.5000E 00	R1= 5.0000E-01	R2(2)= 2.1611E-01	R3= 2.6946E-01
	C1= 3.0000E 00	C2(2)= 4.6273E 00	C3= 5.5667E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5000E 00	R1= 5.0000E-01	R2(1)= 4.1563E-01	R3= 2.6946E-01
	C1= 3.0000E 00	C2(1)= 2.4060E 00	C3= 5.5667E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5000E 00	R1= 1.0000E 08	R2(2)= 4.3221E 07	R3= 5.3892E 07
	C1= 1.5000E-08	C2(2)= 2.3137E-08	C3= 2.7833E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5000E 00	R1= 1.0000E 08	R2(1)= 8.3126E 07	R3= 5.3892E 07
	C1= 1.5000E-08	C2(1)= 1.2030E-08	C3= 2.7833E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 2.0000E 05	R2(2)= 9.2118E 04	R3= 1.0704E 05
	C1= 7.7500E-06	C2(2)= 1.0167E-05	C3= 1.4481E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 2.0000E 05	R2(1)= 1.4041E 05	R3= 1.0704E 05
	C1= 7.7500E-06	C2(1)= 6.6698E-06	C3= 1.4481E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 2.5000E 05	R2(2)= 1.1515E 05	R3= 1.3380E 05
	C1= 6.2000E-06	C2(2)= 8.1332E-06	C3= 1.1585E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 2.5000E 05	R2(1)= 1.7551E 05	R3= 1.3380E 05
	C1= 6.2000E-06	C2(1)= 5.3359E-06	C3= 1.1585E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 1.0000E 06	R2(2)= 4.6059E 05	R3= 5.3518E 05
	C1= 1.5500E-06	C2(2)= 2.0333E-06	C3= 2.8962E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 1.0000E 06	R2(1)= 7.0206E 05	R3= 5.3518E 05
	C1= 1.5500E-06	C2(1)= 1.3340E-06	C3= 2.8962E-06
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01

	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 1.5000E 05	R2(2)= 6.9089E 04	R3= 8.0278E 04
	C1= 1.0333E-05	C2(2)= 1.3555E-05	C3= 1.9308E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 1.5000E 05	R2(1)= 1.0531E 05	R3= 8.0278E 04
	C1= 1.0333E-05	C2(1)= 8.8931E-06	C3= 1.9308E-05
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 5.0000E-01	R2(2)= 2.3030E-01	R3= 2.6759E-01
	C1= 3.1000E 00	C2(2)= 4.0666E 00	C3= 5.7924E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 5.0000E-01	R2(1)= 3.5103E-01	R3= 2.6759E-01
	C1= 3.1000E 00	C2(1)= 2.6679E 00	C3= 5.7924E 00
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 1.0000E 08	R2(2)= 4.6059E 07	R3= 5.3518E 07
	C1= 1.5500E-08	C2(2)= 2.0333E-08	C3= 2.8962E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01
A= 1.5500E 00	R1= 1.0000E 08	R2(1)= 7.0206E 07	R3= 5.3518E 07
	C1= 1.5500E-08	C2(1)= 1.3340E-08	C3= 2.8962E-08
	DCK1= 2.2500E 00	DCK2= 1.0525E 01	DCK3= 1.0300E 01
	DK1= 2.2500E 00	DK2= 1.0525E 01	DK3= 1.0300E 01

APPENDIX III

PROGRAM DRAWCOMP

```
      PROGRAM  DRAWCCMP
      DIMENSION DRAWR1(900),DRAWR21(900),DRAWR22(900),
     1DRAWR3(900),DRAWC1(900),DRAWC21(900),DRAWC22(900),DRAWC3(900),
     2DRAWA(900),ITITLE(12)
      DO 999 J=1,900
      DRAWR1(J)=0.0
      DRAWR21(J)=0.0
      DRAWR22(J)=0.0
      DRAWR3(J)=0.0
      DRAWC1(J)=0.0
      DRAWC21(J)=0.0
      DRAWC22(J)=0.0
      DRAWC3(J)=0.0
  999 DRAWA(J)=0.0
      READ 50,DK1,DK2,DK3,STARTA,STEPA,STOPA
   50 FORMAT(6F10)
      READ 60,R1
   60 FORMAT(F10)
      A=STARTA
      I=1
  100 Y=DK2/A-A-(2.*DK1)/(A**2)
      IF(Y) 101,101,102
  101 A=A+STEPA
      IF(STOPA-A)125,125,100
  102 X=DK3-2.*A-DK1/(A**2)
      IF(X) 101,101,103
  103 T13=X-Y
      IF(T13)101,101,104
  104 W=(Y**2)-(4.*DK1*T13)/(A**2)
      IF(W) 101,105,105
  105 CONTINUE
      C3=T13/R1
      C1=A/R1
      R3=A/C3
      R21=(Y+SQRTF(W))/(2.*C3)
      C21=DK1/((A**2)*R21)
      R22=(Y-SQRTF(W))/(2.*C3)
  106 C22=DK1/((A**2)*R22)
      DRAWR1(I)=R1
      DRAWR21(I)=R21
      DRAWR22(I)=R22
      DRAWR3(I)=R3
      DRAWC1(I)=C1
      DRAWC21(I)=C21
      DRAWC22(I)=C22
      DRAWC3(I)=C3
      DRAWA(I) =A
      I=I+1
      GO TO 101
  125 CONTINUE
      NUM=498
      DO 700 JK=1,12
  700 ITITLE(JK)=8H
      ITITLE(1)= 8HLILLIS.
      ITITLE(2)= 8H J. W.
      ITITLE(3)= 8H THESIS
```

```
      ITITLE(4)= 8H   PLOT
      ITITLE(5)= 8HOF R2(1)
      ITITLE(6)= 8H, R2(2),
      ITITLE(7)= 8H AND R3
      ITITLE(8)=8HVS TIME
      ITITLE(9)= 8HCONSTANT
      ITITLE(10)=8H, A, FOR
      ITITLE(11)=8H CONSTAN
      ITITLE(12)=8HT R1
      LABEL=4HR21
      CALL DRAW(NUM,DRAWA,DRAWR21,1,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      LABEL=4HR1
      CALL DRAW(NUM,DRAWA,DRAWR1,2,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      LABEL=4HR22
      CALL DRAW(NUM,DRAWA,DRAWR22,2,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      LABEL=4HR3
      CALL DRAW(NUM,DRAWA,DRAWR3,3,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      DO 701 JK=5,12
  701 ITITLE(JK)=8H
      ITITLE(5)= 8HOF C1,C2
      ITITLE(6)= 8H(1),
      ITITLE(7)= 8HC2(2), A
      ITITLE(8)= 8HND C3 VS
      ITITLE(9)= 8H TIME CO
      ITITLE(10)=8HNSTANT,
      ITITLE(11)=8HA, FOR C
      ITITLE(12)=8HONST, R1
      LABEL=4HC22
      CALL DRAW(NUM,DRAWA,DRAWC22,1,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      LABEL=4HC1
      CALL DRAW(NUM,DRAWA,DRAWC1,2,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      LABEL=4HC21
      CALL DRAW(NUM,DRAWA,DRAWC21,2,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      LABEL=4HC3
      CALL DRAW(NUM,DRAWA,DRAWC3,3,0,LABEL,ITITLE,0,0,0,0,
     10,0,0,0,0,LAST)
      END
```

APPENDIX IV

CONTINUANT EXPANSIONS

It was previously stated that the expansion of a continuant can be formulated as the evaluation of a signal-flow graph representation based on Euler's rule. The method is especially useful when working with RC ladder networks since all the b coefficients of the continuants are unity. The signal-flow graph for the network would be as shown in Fig. 11.

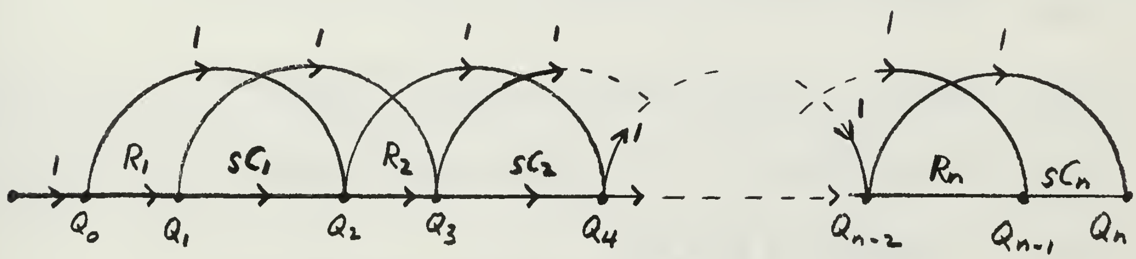


Fig. 11. Flow Graph of a Continuant

Rules for evaluation of the signal-flow graph are as follows:

1) The first term is given by the product of all adjacent R and sC transmissions.

2) The second term is given by the sum of products formed from the first term by omitting adjacent pairs of R and sC transmissions, taken one pair at a time, in every possible way. (This is equivalent to setting the transmission of the omitted pair equal to unity.)

3) The third term is given by the sum of products formed from the first term by omitting adjacent pairs of R and sC transmissions, taken two pairs at a time, in every possible way.

4) The nth term is given by the sum of products formed from the first term by omitting adjacent pairs of R and sC transmissions, taken (n-1) pairs at a time, in every possible way.

In forming the sums of products specified in the above four rules, it is convenient to think of the omissions of a transmission of an adjacent R and sC pair as being equivalent to "bridging around" this pair with a path of unity transmission.

The continuant expansions of

$$Q_m(s) = k(R_1, sC_n)$$

for values of m and n up to m = 8 and n = 4 are as follows:

$$Q_0(s) = 1$$

$$Q_1(s) = R_1$$

$$Q_2(s) = (R_1C_1)s + 1$$

$$Q_3(s) = (R_1C_1R_2)s + (R_1 + R_2)$$

$$Q_4(s) = (R_1C_1R_2C_2)s^2 + (R_1C_1 + R_1C_2 + R_2C_2)s + 1$$

$$Q_5(s) = (R_1C_1R_2C_2R_3)s^2 + (R_1C_1R_2 + R_1C_1R_3 + R_1C_2R_3 + R_2C_2R_3)s + (R_1 + R_2 + R_3)$$

$$Q_6(s) = (R_1C_1R_2C_2R_3C_3)s^3 + (R_1C_1R_2C_2 + R_1C_1R_2C_3 + R_1C_1R_3C_3 + R_1C_2R_3C_3 + R_2C_2R_3C_3)s^2 (R_1C_1 + R_1C_2 + R_1C_3 + R_2C_2 + R_2C_3 + R_3C_3)s + 1$$

$$
\begin{aligned}
Q_7(s) &= (R_1C_1R_2C_2R_3C_3R_4)s^3 + (R_1C_1R_2C_2R_3 + R_1C_1R_2C_2R_4 \\
&+ R_1C_1R_2C_3R_4 + R_1C_1R_3C_3R_4 + R_1C_2R_3C_3R_4 + R_2C_2R_3C_3R_4)s^2 \\
&\quad + (R_1C_1R_2 + R_1C_1R_3 + R_1C_1R_4 + R_1C_2R_3 + R_1C_2R_4 + R_1C_3R_4 \\
&\quad + R_2C_2R_3 + R_2C_2R_4 + R_2C_3R_3 + R_3C_3R_4)s + (R_1 + R_2 + R_3 + R_4)
\end{aligned}
$$

$$
\begin{aligned}
Q_8(s) &= (R_1C_1R_2C_2R_3C_3R_4C_4)s^4 + (R_1C_1R_2C_2R_3C_3 + R_1C_1R_2C_2R_3C_4 \\
&\quad + R_1C_1R_2C_2R_4C_4 + R_1C_1R_2C_3R_4C_4 + R_1C_1R_3C_3R_4C_4 \\
&\quad + R_1C_2R_3C_3R_4C_4 + R_2C_2R_3C_3R_4C_4)s^3 + (R_1C_1R_2C_2 \\
&\quad + R_1C_1R_2C_3 + R_1C_1R_2C_4 + R_1C_1R_3C_3 + R_1C_1R_3C_4 + R_1C_1R_4C_4 \\
&\quad + R_1C_2R_3C_3 + R_1C_2R_3C_4 + R_1C_2R_4C_4 + R_1C_3R_4C_4 + R_2C_2R_3C_3 \\
&\quad + R_2C_2R_3C_4 + R_2C_2R_4C_4 + R_2C_3R_4C_4 + R_3C_3R_4C_4)s^2 \\
&\quad + (R_1C_1 + R_1C_2 + R_1C_3 + R_1C_4 + R_2C_2 + R_2C_3 + R_2C_4 + R_3C_3 \\
&\quad + R_3C_4 + R_4C_4)s + 1
\end{aligned}
$$

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library U. S. Naval Postgraduate School, Monterey, California	2
3.	Commander, Air Systems Command (Code AIR-09B) Department of the Navy Washington, D. C. 20360	1
4.	Prof S. R. Parker (Thesis Advisor) Department of Electrical Engineering Naval Postgraduate School, Monterey, California	1
5.	Lt Jack Warner Lillis, USN 1135 Leahy Road Monterey, California	1

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Ranges of Allowable Component Values for the Synthesis of Specified RC Transfer Functions

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Masters thesis, June 1967

5. AUTHOR(S) (Last name, first name, initial)

Lillis, Jack W., Lieutenant, United States Navy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1967	92	6

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

[redacted]

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY

13. ABSTRACT

The theory of continuants is applied to the analysis of general ladder networks of the first Cauer form to provide a concise, compact, and readily calculable form for the driving point and transfer functions used to describe the network. A new procedure is established for the synthesis of RC ladder networks up to nth order from a given voltage-ratio transfer function. Ranges of values for the network components are shown to exist. The procedure is readily adaptable for use with a digital computer. Two programs written in FORTRAN 63 language are provided for a third order system to illustrate how the procedure can be used with a digital computer for computer-aided design of the networks. Output of the programs is in the form of graphs and tables of component values.

DD FORM 1 JAN 64 1473

Unclassified

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
SYNTHESIS/NETWORKS/LADDER/RC/FUNCTIONS/TRANSFER						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

A-31409